Integrated Quality and Maintenance Models For Multistage Production Systems With Batch Shipments

BY

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This thesis, written by **SYED ALI ZAMIN** under the direction of his Thesis Advisor and approved by his Thesis Committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN SYSTEMS ENGINEERING.**

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Dedicated

to

my beloved parents and my brother Abid,

whose presence relieved me from many worries and whose absence is felt all the time.

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THESIS ABSTRACT

Name: SYED ALI ZAMIN

Title: Integrated Quality and Maintenance Models For Multi-Stage Production Systems with Batch Shipments

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Multi-stage production inventory systems are very common in a manufacturing environment. Production, quality and maintenance are three important and inter-related functions in any industrial process. These aspects are traditionally treated as separate problems. In this thesis we integrate quality, restoration and maintenance aspects to some multi-stage lot sizing models with batch shipments. The optimal number of batches and lot sizes are determined so that the total cost is minimized. The first model deals with a multi-stage production inventory system with batch shipments between stages. The second model considers integrated vendor buyer problem where the buyer faces deterministic demand. The third model also deals with integrated vendor buyer system but the buyer faces stochastic demand. Several numerical examples are used to demonstrate the usefulness of the developed models.

Keywords: Multi-Stage Production Systems, batch shipment, Integrated Vendor Buyer relationship, Quality, Restoration and Maintenance.

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خلاصة الرسالة

الاسم : سيد علي زامن

العنوان : "نماذج متكاملة للجودة والصيانة في أنظمة الإنتاج متعدد المراحل ذات التوصيل بالدفعات"

التخصص : هندسة النظم

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تعد أنظمة المخزون ذات الإنتاج متعدد المراحل شائعة في بيئات التصنيع. تعتبر وظائف الإنتاج والجودة والصيانة مهمة ومترابطة في أي عملية صناعية. عادة يتم التعامل مع هذه الوظائف باعتبارها عناصر منفصلة. في هذا البحث تتكامل كل من الجودة والصيانة في أنظمة الإنتاج متعدد المراحل لتحديد حجم الشحنة وعدد الدفعات. يتم تحديد العدد الأمثل للدفعات والحجم الأمثل للشحنة بحيث تقل التكلفة الكلية. يعنى النموذج الأول بنظام مخزون ذي إنتاج متعدد المراحل وإيصال بالدفعات بين المراحل. النموذج الثاني نموذج متكامل للبائع والشاري حيث يكون طلب الشاري معلوماً تحديداً. النموذج الثالث يعنى بنظام البائع والشاري حين يكون طلب الشاري احتمالياً. استخدمت العديد من الأمثلة العددية لتبيين أهمية هذه النماذج.

درجة ماجستير في العلوم

جامعة الملك فهد للبترول والمعادن

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Chapter 1

Introduction

1.1 General Background

Most inventory systems encountered in the real world are multi-echelon in nature, which means they have more than a single stocking point. These inventory systems are also termed as Multi-Stage Production inventory systems ($MS - PIS$), where each stage receives its input from previous stage and supplies its output to the later stages. The stocking or inventory in MS-PIS is a major contributor to poor quality and longer cycle time. Because inventory that is held for future use often hides many quality problems, with the aging of inventory and larger lot size proper feed back is masked by days, weeks and months that have gone by. Thus, the harder it is to correct quality problems because the data trail is stale. In addition inventory causes excessive material handling which contributes to cost and does not add value to the product.

For a breakthrough attention has shifted from increasing efficiency by means of the so called economies of scale and interval specialization to meeting market condition in terms of flexibility, delivery performance and quality. This is a trend towards *Just In Time (JIT)* production which implies working without inventories at all. Without JIT, cycle time reduction cannot be dramatically impacted. It also requires total commitment and good coordination between different stages. JIT is a Japanese Management technique for inventory control and materials management that has as its aim the complete elimination of waste including unnecessary inventory and scrap in production. It can also be described as a quality and scrap control tool, as a stream lined plant configuration that raises process yields, as a production line balancing approach, and as an employee involvement and motivational mechanism. The overall goal is to reduce inventories to as close to zero as possible by producing only enough work units to keep the next work station in a production process in operation. The JIT concept has the following advantages.

1. Shortened lead time.

2. Reduced time spent on non process work.

3. Reduced inventory.

4. Better balance between different processess.

A *supply chain* is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers. The inventory decision is one of the major decisions in supply chain management. Most researchers have approached the management of inventory from an operational perspective. These include deployment strategies, control policies, determination of the optimal levels of reorder quantities and reorder points, and setting safety stock levels at each stocking location.

The basic idea behind *Just in Time* philosophy and *Supply chain management* is to have the least WIP (Work in Process) inventory. These inventories exist at every stage of the production system as either raw materials, semi-finished or finished goods. A number of different models have been developed in which the batches were shipped to the next stage in order to have smaller inventory, but they all consider perfect production process which is not realistic, especially when having inventory so low that any unplanned unavailability of means of production will directly result in serious delivery problems. This fact has shifted the focus to maintenance and the need for efficient maintenance policies which are effective tools to cut costs. When the process is in a good condition, the items produced would be of high or acceptable quality; but with the passage of time the process may shift to a stage in which it starts producing non conforming items. To avoid or, at least delay the shift of the process from an in control state to an out of control state preventive maintenance procedures are adopted. In general the sooner we realize that the process is out of control, the less costly it would be to repair and restore it. Preventive maintenance is the action taken prior to the to the incidence of failures to avoid associated costs.

The preventive maintenance activities have the dual effect of reducing the amount of nonconforming items produced and the cost of restoring the machines back to the in control state.

In literature there is only a very limited number of multi stage lot size models which deal with simultaneous optimization of the production lot size and the corresponding transportation batch sizes, the inter-stage interaction and existence of work in process inventories in MS-PIS (Multi-Stage Production Inventory Systems) with batch shipment complicates the issue. That is why the models that are developed with this concept considered perfect production process. However, integrating quality aspects in a multi stage production model presents additional practical complications. The following important issues must be addressed when dealing with imperfect multi-stage production processes.

- Nonconforming items must be screened so that they are not passed to subsequent stages to avoid waste of resources and unnecessary processing.
- While screening non-conforming items, errors may be committed. Non-conforming items may be incorrectly accepted and good items incorrectly rejected.
- Process shift to an out of control state where non-conforming items are produced must be prevented or at least delayed, and detected quickly if it occurs.

The purpose of this thesis is to integrate quality and maintenance considerations into the models related to multi-stage production system and investigate their effect. Literature survey will be done in the area of multi-stage lot sizing production inventory system, where batches are transferred to the next stage during the production, and also the area of joint vendor buyer inventory models. Quality, preventive maintenance and restoration aspects will be incorporated into some recent multi stage models having lot streaming and some vendor buyer inventory policies.

1.2 Problem Definition

More and more companies are shifting towards *Just In Time* and *Supply Chain Management* where the production cost is minimized by minimizing the WIP inventories. The vendor buyer inventory models in supply chain management can be considered as a special case of multi stage production having two stages where the production of the first stage (vendor) is always greater then the production rate (or demand) of the second stage (buyer). In this section some basic models, i.e [1], [2] and [3], which will be the basis for the work are discussed briefly (as they are explained in Chapter (3), Chapter (4) and in Chapter (5) completely) . In multi-stage production system, transferring of bad items to the next stage will result in unnecessary cost, that is why it is assumed that they are screened out before shipping to the next stage. The product is also assumed to be infinitely divisible, thus lot sizes are not required to be integers. These models have been published recently. They are motivated by recent development in *Just in Time* system and *Supply chain management.* However all of these models assumed perfect quality and perfect production processes.

We develop generalized integrated models incorporating quality, maintenance and restoration into the batch sizing decisions and their numbers in the context of MS-PIS with batch shipments. We consider two cases. i.e. when having more than two stages and a special case of having two stage production system which can be called as integrated vendor-buyer relationship.

The first Model (Model 1) is of lot streaming for a multi stage process. The second model (Model 2) considers a problem of a vendor supplying a product to a buyer having a deterministic demand. The third Model (Model 3) is similar to the second model, but the buyer uses a continuous review inventory policy facing a stochastic demand.

1.2.1 MS-PIS with Batch shipment

We work on the model given by Ronald et al. [1] which considered deterministic demand, by incorporating quality and restoration in it. The model has n stages and $m_j, (j = 1, \cdots, n)$ number of batches in each stage. The batch sizes in each stage follow an increasing or decreasing geometric series depending upon the production rate. (i.e.. if $P_j > P_{j+1}$ then batches will be produced in the order of decreasing geometric series and vice versa). In our model we incorporate the quality and restoration to the basic model in order to get the generalized mathematical model for total expected cost which will consists of

1. Inventory Holding Cost.

2. Setup and Transportation cost.

3. Quality related cost.

4. Restoration cost.

1.2.2 Integrated Vendor-Buyer relationship

We worked on the model given by Goyal et al. [2] and by BenDaya et al. [3], by adding quality and restoration aspects.

The model given by [2] assumed perfect production process with deterministic demand rate, in which the vendor first shipped a small batch and the rest of the shipments are the product of the first shipment and the ratio of production and demand. We worked on it by integrating quality and restoration aspects. The total cost of the developed model was the sum of Inventory Holding cost, Setup and transportation cost, cost of producing nonconforming items and restoration cost.

The model given by [3] considered continuous review inventory policy with lost sales, where the the process is assumed to be perfect and the buyer faced stochastic demand, the vendor shipped the items to the buyer in batches of equal size. The total cost of the developed model consists of;

1. Inventory holding cost (incurred by both vendor and buyer).

2. Ordering and transportation cost (for buyer).

3. Back-ordering or cost due to lost sale (for buyer).

4. Setup cost (for vendor).

5. Quality related cost (incurred by vendor).

6. Restoration cost (also incurred by vendor).

1.2.3 Thesis Objectives

To develop and test mathematical models integrating production, quality and maintenance aspects for series MS-PIS with batch shipments. The following are the objectives of the thesis:

1. Literature survey will be done in the area of multi stage lot sizing production inventory systems, where batches are transferred to the next stage during the production, and also the area of joint vendor buyer inventory models.

2. Incorporating quality and restoration aspects into the multi stage model proposed by [1] having a deterministic demand and lot streaming.

3. Incorporating quality, preventive maintenance and restoration aspects into the model proposed by [2].

4. Incorporating quality, maintenance aspects into the model proposed by [3] assuming stochastic demand for the buyer.

1.2.4 Thesis Organization

This Thesis presentation is organized in six chapters, Chapter 2 presents a comprehensive survey of the relevant literature. Chapter 3 presents the development of integrated production, quality and restoration model of a series MS-PIS with batch shipment. Chapter 4 presents the development of integration of production, quality and restoration in integrated vendor-buyer relationship with deterministic demand, and Chapter 5 presents the integration of production, quality and restoration in the integrated vendor-buyer relationship with stochastic demand. Chapter 6 concludes the thesis.

Chapter 2

Literature Review

2.1 introduction

A multi-stage production inventory system (MS-PIS) with batch shipment is one in which the each stage receives its input in batches from one or more immediate predecessors and supplies its out put to one or more immediate successors in batches. MS-PIS are, in essence the most common case in any manufacturing environment. The purpose of this chapter is to review literatures which cover two areas of research: multi stage production inventory systems with lot streaming (batch shipment), and joint vendor buyer production inventory models.

2.2 Review of Multi-echelon inventory systems

The control of process inventory and especially its functional relationship with the cycle time has received a lot of attention in recent years. The larger the production lot size the longer the manufacturing cycle which in turn increases the process inventory. Different lot size models for multi echelon inventory systems present in the literature include many variations in detail.

Koeningsberg [4] reviewed the basic problems associated with efficient operations of production lines. Clark [5] analyzed literature on multi echelon inventory problems. Aggarwal [6] reviewed the lot sizing techniques under stationary demand conditions, with emphasis on pure inventory situations, and also discussed the literature on MS-PIS. The literature on management of production inventories are reviewed and a comparison of material requirements planning and statistical inventory control has been made by Fortuin [7]. Silver [8] looked at the existing theory in inventory management. De Bodt et al. [9] examined the lot sizing under dynamic condtitions in production inventory situations. Chikan [10, 11] presented a review of inventory theory and models. Chikan [12] gave comparison of various classification systems of inventory models. Clark [13] presented an account of events leading to one of the earliest studies of multi echelon production inventory systems by Clark et al. [14]. Subsequent extension are mentioned and areas of current and future applications are discussed. Eilon [15] presented a classical economic production quantity (EPQ) model that optimizes the return on production cycle by determining the length of the production cycle. Taha and Skeith [16] recognized the relationship between manufacturing cycle time and the cost of holding WIP inventories in developing a model for a single product multi-stage production system. Banerjee et al. [17] developed a general approach for tackling the lot sizing problems in single stage and multi stage systems taking into account work in process inventories and, more importantly, incorporating the notion of gradual transformation of input to output at every stage.

In the literature there is only a very limited number of multi stage lot size models which deal with the simultaneous optimization of the production lot size and the corresponding transportation batch size. A fundamental shortcoming of most conventional lot size models is that they completely ignore the fact that produced items have to be transported between consecutive stages. It seems to be more realistic in a multi stage production system to allow transports of partial lot or batches between the stages, where the optimal number and size of the batches can differ between adjacent stages.

We will concentrate mainly on lot sizing models for multi echelon production inventory system with batch shipment for a single product under the condition of finite production rate. Two main classes that can be identified are as follows;

1. The *MS-PIS with batch shipment*, where the demand is deterministic and in which a uniform lot size is produced through all stages with a single setup and without interruption at each stage. Partial lots called batches may be transported to the next stage upon completion in order to minimize cycle

time which results in lower WIP.

2. The *Integrated vendor buyer relationship*, for both deterministic as well as stochastic demand, where the items are shipped in batches to the buyer in order to minimize the inventory related cost.

2.2.1 Review of Lot sizing models for MS-PIS with Batch shipment

Szendrovits [18] had presented one of the earliest model in which shipment of equal sized batches were allowed over the stages assumed sunk transportation cost or without charging additional transportation cost. The model assumed that a uniform lot size is manufactured with only one setup at each stage. Linear inventory holding costs and a constant and continuous demand of finished products are assumed over an infinite horizon. He showed that the resulting overlapping between operations at the succeeding stages reduces the manufacturing cycle time as well as the total costs of a given production lot size.

Goyal [19] noted that both the EPQ and the optimal number of transports can be calculated simultaneously by a very similar model discussed by [18]. In this model Goyal considered the effect of sub batches on EPQ and suggested the form of transporation cost function, with fixed costs per transport through all stages.

Szendrovits [20] developed a simple and faster computational procedure for solving the extension done in [19].

Szendrovits and Drezner [21] allowed a varying number of equal size batches at different stages. A low transportation cost at a certain stage would permit a larger number of batches then at another stage with a higher transportation cost.

Goyal [22] explored the effect of the number of sub-batches on the economic batch quantity, and gave a more general expression for determining the average investment in process inventory.

Goyal [23] gave a model for two stage production system and advocated the use of unequal batch sizes that follow an increasing or decreasing geometric series. His approach resulted in cost lower then those obtained by equal sized batches. However this model is limited to two stage system.

Scendrovits [24] has shown that additional transportation of those unequal batch sized that exceeds the load capacity of the transport equipment may well eliminate the savings in inventory costs.

Szendrovits and Golden [25] examined two models in which one model uses uniform lot sizes at all stages and allows transportation of equal sized portions of the lot between stages. The other model allows a different lot size at each stage, so that only complete lots are transported to the next stage. The costs of the two models were then compared, which generally shows that the model in which batch shipment is allowed results in yielding significantly lower costs.

Drezner, Szendrovits and Wesolowsky [26] considered a model where lots may be of different sizes. In addition, either completed lots or partial lots called batches may be transported to succeeding stages. They developed a heuristic solution procedure for their model.

Szendrovits [24] noted that the transportation costs of batches are in general related to a certain load capacity, and that the effect of the relaxed batch size constraint on transportation costs was ignored by [23].

Goyal and Szendrovits [27] presented a model with limited transportation capacities and an unrestricted number of stages. They allowed both the shipment of equal and unequal sized batches between the stages, in addition to different number of batches at any stage in their model. The authors presented a heuristic procedure for solving this optimization problem.

Szendrovits [28] developed a model with equal sized batches, where keeping the considered facility setup is allowed while production is interrupted temporarily. He added a penalty cost for idle time.

Bogaschewsky, Buscher and Lindner [1] developed a modification of the unrestricted two stage model proposed by Goyal [23] for a multi-stage manufacturing system, where the partial lots called batches may be transported to the next stage upon completion. The number of unequal sized batches may differ across stages. Both the same and different number of batches transported to the next stage is considered. Optimal solutions were given for both cases.

Ramasesh [29] presented an economic production lot sized model by considering transportation time, waiting time, setup time and processing time. The sub-lots (batches) transported to the next stage were of equal size.

2.2.2 Review of Integrated Vendor-Buyer models

There is quite a few literatures available where the vendor's and the buyer's inventory problem are treated separately. However the literature dealing with the interaction between the buyer and the supplier of an item is in its infancy stage. The literature which deals with integrated models could be classified in the following way;

1. models which deal with joint economic lot sizing policies.
2. models which deal with coordination of inventory by simultaneously determining the order quantity of the buyer and the vendor.
3. models which deal with integrated problem but do not determine simultaneously the order quantity of the buyer and the vendor.
4. models which deal with buyer -vendor coordination due to marketing considerations.

Since the basic model deal with Joint Economic Lot sizing JELS policy, so here we will only discuss the literature that are related to the that policy.

Joint Economic Lot Size Models

It consists of lot size of formulas based on the joint optimization of vendor and purchaser where the objective is to minimize the total inventory related costs. here we will discuss the models considering both deterministic as well as stochastic demand.

With Deterministic Demand

Goyal [30] suggested a joint economic lot size model where the objective is to minimize the total relevant cost of both vendor and buyer. He assumed an infinite production rate for the vendor (i.e. the vendor does not manufacuter the items himself but in turn buys it from his vendor).

Lee et al. [31] pointed out that as quantity discounts are introduced in [30], the inventory holding costs are no longer remain constant.

Banerjee [32] for the first time coined the term JELS and investigated the cost trade offs involved in adopting the JELS from both the purchaser's and vendor's point of view. The cost consists of annual holding costs (for both vendor and buyer), setup and unit production cost (for vendor), and ordering and unit purchase cost (for buyer). He assumed finite production rate and examined a lot for lot model in which the vendor manufactures each buyer shipment as a separate batch.

Goyal [33] further generalized [32] by relaxing the assumption of the lot for lot policy of the vendor and showed that his JELS model provided a lower or equal joint total relevant cost as compared to [32] by using an equal shipment policy.

Affisco et al. [34] investigate the concepts of JELS and vendor setup cost reduction for the case of single vendor and single purchaser and assumed a logarithmic investment function.

Affisco et al. [35] then extended their model [34] to the case of one vendor and many non identical purchasers.

Nasri et al. [36] investigated the impact of simultaneous investment in setup cost and order reduction on Banerjee's [32] (JELS) model.

Lu [37] gave an optimal solution for the case of single vendor single buyer and developed an iterative Heuristic approach for single vendor and multi buyers case.

Goyal and Gupta [38]reviewed the integrated inventory models (buyer-vendor coordination).

Goyal [39] developed a JELS model and used a shipment policy where the size of the batches increased by a fixed factor equal to the production rate divided by the demand rate and showed that this shipment policy gave lower cost .

Hill [40] took the idea given by [39] a stage further by considering a general successive shipment size policy, of which the equal shipment size policy [33] and Goyal's [39] policy represent special cases. He defined a factor which is the proportional increase in the size of successive shipments and had range in $[1, P/D]$. Although the cost is lower then either of the special cases, but because of rather complicated nature of the shipment sizes provided by the solution means that they are of analytical rather then of immediate practical interest.

Affrisco, Paknejad and Nasri [41] compared JELS and IRRD (individually responsible and rational decision) policies and suggested that when an environment of cooperation between the parties has been established the JELS is superior policy.

Aderohunmu, Mobolurin and Bryson [42] discussed equal shipment size policy.

Hill [43] determined the form of the globally optimal batching and shipping policy which was the combination of [39] and [33] and is some what similar in structure to [27]. Although this policy is giving the lower cost solutions, but this is at the expense of producing solutions that are complicated and not practical.

Goyal [2] considered a problem of determining economic production and shipment policy of a product supplied by a vendor to a single buyer, and developed a simple alternative policy. Goyal showed that this method is simpler and very often achieves cost near to the optimal one presented by [43].

With Stochastic Demand

Although for deterministic system there is no difference in periodic review inventory policy and the continuous review inventory policy or (s, Q) policy, but the nature of the two typed of models becomes somewhat different for stochastic demands.

In *periodic review inventory policy* the state of inventory system is examined only at discrete, usually equally spaced points in time. Decisions concerning the operation of the system such as whether or not to place an order are made only at these review times. In fact, the decision maker knows nothing about the state of the system at times other than the review times.

In *Continuous review inventory policy or* (s, Q) *policy* each transaction (demand, placement of order, receipt of shipment etc.) is recorded and reported as they occur, and the information is immediately made know to the decision maker. When this type of policy is used, then it is possible to make decisions concerning the operation of the system, such as the decision as to whether or not to place an order each time a demand occurs.

In this review we shall confine our attention to the models that are dealing with *continuous review inventory policy*.

In recent years, there have been several research studies in the area of inventory control to provide more realistic, practical and useful mathematical models to decision makers. The continuous review inventory policy or the (s, Q) inventory models with lost sales was first discussed by **Hadley and Whitin** [44]. They derived an exact formulation of the average inventory cost for an (s, Q) policy with Poisson demand and constant deterministic lead times. Under (s, Q) policy, an order is placed to increase the inventory position to Q as soon as the inventory position drops to or below s.

More recently **Johansen and Thorstenson** [45] formulated and solved the same model as a semi-Markov decision model.

Sivazlian [46] considered a continuous review (s, Q) policy, with the assumptions of unit demand arrivals and general demand inter arrival times.

Richards [47] extended this result to the case with random demand size.

Liao and Shyu [48] presented a probabilistic inventory model and stated that there is a lack of an appropriate inventory model that treats lead time as decision variable. The authors have presented a model which can be used to determine the length of lead time that minimized the expected total relevant cost.

BenDaya and Abdul Raouf [49] first extended the model given by Liao et al. [48] in which they considered both the lead time and the order quantity as decision variable, then proposed models which use different relationship between lead time crashing cost and lead time. They assumed that the demand follow a normal distribution.

Ouyang et al. [50] proposed a continuous review inventory model with lead time reduction by allowing shortages with a mixture of back orders and lost sales. **Moon and Choi** [51] and **Hariga and BenDaya** [52] improved and revised Ouyang [50] model by considering the reorder point as one of the decision variables, they further developed a minimax distribution free procedure for the problems. These papers are focusing on the benefits from lead time reductions in which ordering cost is treated as a fixed constant. Later **Ouyang et al.** [53] investigated the impact of ordering cost reduction on the modified continuous review inventory systems involving variable lead time with a mixture of back orders and lost sales.

Most mathematical models encountered in the inventory literature dealing with the stochastic continuous review (s, Q) model, assumed that lead time is a given problem parameter, **Kim and Benton** [54] questioned this unrealistic assumption and considered the effect of lot size on lead time and safety stock. They established a linear relationship between lead time. Later **Hariga** [55] modified the model in [54] by developing the relationship between lot size and lead time and presented an effective iterative procedure that determines the optimal or near optimal lot size and reorder point that minimize the total annual inventory cost for the model.

Ben-Daya and Hariga [3] then extend the model given by [55] and developed a (s, Q) policy where the buyer uses a continuous review inventory policy, and the items are shipped to the buyer in batches of equal size in order to minimize the inventory cost.

Chapter 3

MS PIS with Batch Shipment

The purpose of this chapter is to model the effect of imperfect production processes on batch sizing and their numbers in the context of uniform lot size multi-stage production system where batches are allowed to be shipped under deterministic condition.

3.1 Overview

Determining economic lot sizes has been of major interest in theory, where a huge number of different models has been developed. The number of items produced without change-over defines the lot size. The EPQ model balances the holding and setup cost in order to minimize the total cost.

In Multi-Stage production system considering Fig. 3.1, the stocking of items at many echelons or stages contributes to poor quality and longer cycle time which is the main contributor of high inventory cost. This had shifted the attention from increasing efficiency by means of the so called economies of scale and internal specialization to meeting market condition in terms of flexibility, delivery performance and quality. This lead towards Just In Time (*JIT*) production whose ultimate goal is to produce small lot sizes with good quality products.

The issue of modeling the effect of imperfect production processes on lot sizing decisions does not seem to have been adequately addressed in the literature for MS-PIS with batch shipments. As all of the models assumed that the quality of produced items is perfect and the production processes involved never fail during a production run. Also, the presence of imperfect production processes in an MS-PIS framework calls for screening of non conforming items in order to prevent unnecessary processing at the subsequent stages and reduce waste. Maintenance procedures including corrective and preventive maintenance and restoration should be used to reduce quality cost. Studies from Rosenblatt and Lee [56, 57] and Porteus [58], have shown that for the imperfect case production processes the optimal EPQ was smaller than that determined by classical EMQ model. Obviously, a larger lot size requires a longer production cycle, and hence likely to contain more defective items. BenDaya and Rahim [59] developed a lots sizing model for a multi stage system that addresses the issues of screening of nonconforming items so that they are not passed to subsequent stages to avoid waste of resources and unnecessary processing and committing errors (accepting bad and rejecting good items) while screening nonconforming items.

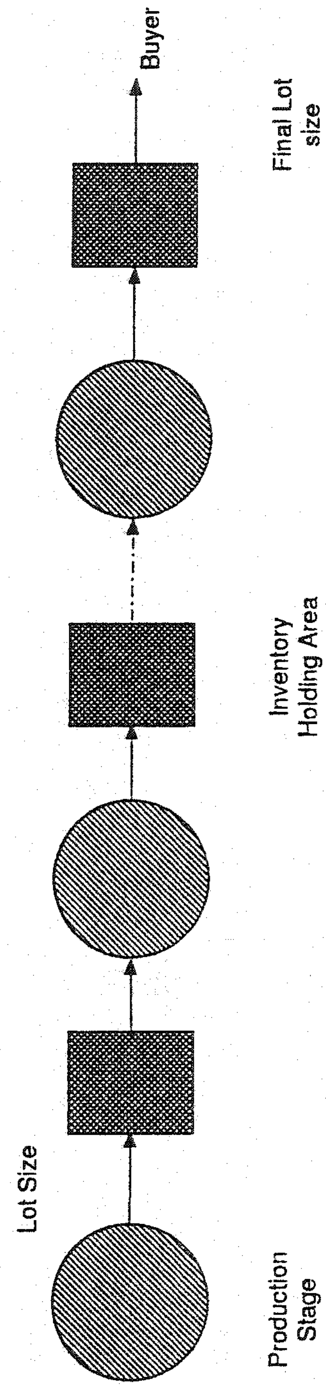

Figure 3.1: Inventory Flow Diagram for a MS-PIS

3.1.1 Introduction of the Basic Model

Bogaschewsky et al [1], considers a model for a multi stage production inventory system in which a uniform lot size is produced through all stages with a single setup and without any interruption at each stage. Partial lots, called batches, may be transported to the next stage upon completion. The number of the unequal sizes batches may differ across stages. Considering setup costs, inventory holding costs, and transportation costs, an optimization method is developed to determine the economic lot size and the optimal batch sizes for each stage. Figure 3.2 illustrates the inventory between two adjacent stages j and $j+1$ when $P_j < P_{j+1}$ and the number of batches equals 3. The upper part shows the inventory for three unequal batches separately, whereas the lower figure shows the inventory profile of the stage. The batch sizes follow a decreasing geometric series (or increasing if $P_j > P_{j+1}$).

Assumptions

The assumptions are described as follows;

1. All parameters are constant and deterministic within the planning period.

2. No backlogging is permitted.

3. A uniform lot size is produced throughout the stage.

4. The transportation of batches to the following stage is allowed before the whole lot is completed at the respective stage.

5. Each lot is produce at each stage with only setup and without interruption.

6. The batch sizes between adjacent stages follow a geometric series.

7. Setup and transportation times are considered insignificant and hence ignored.

8. The rate of continuous demand at sales is lower then the lowest manufacturing rate for a product type through all stages.

9. Every facility is used only for one product at a time.

Notation

Variables

Q = The lot size.

n = Last stage of the production system.

m_j = Total number of batches at stage j.

$q_{i,j}$ = Size of the batch number i in ascending order at stage j .

$q_{min,j}$ = Size of the smallest batch size at stage j.

= $q_{1,j}$

Parameters

P_j = Constant production rate at stage j.

T = Length of the planning period.

D = Total demand in the planning period.

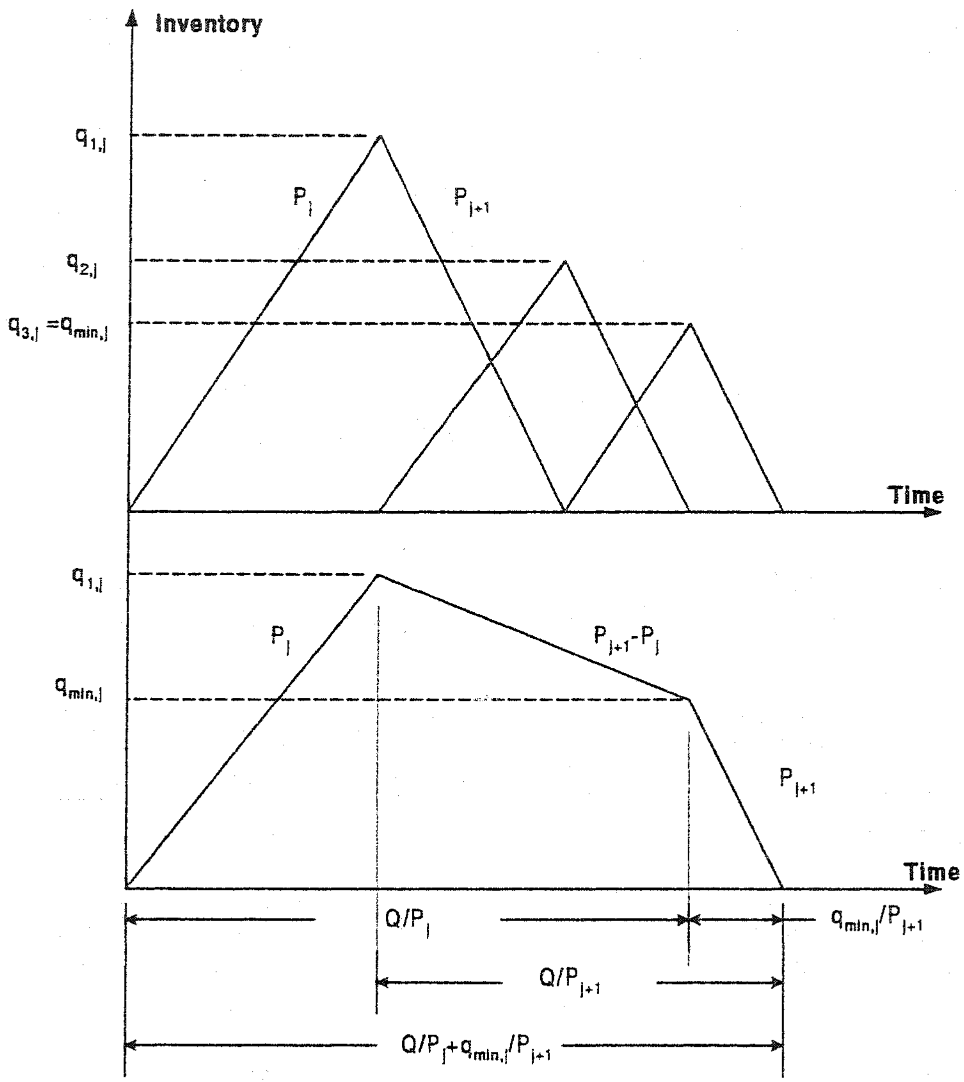

Figure 3.2: Inventory profile for a perfect process (when $P_j < P_{j+1}$ & $m_j = 3$)

h_j = Unit inventory holding cost per unit of time at stage j.

S_j = Setup cost per lot at stage j.

T_j = Transportation cost of one batch from stage j to stage $j+1$.

Since the batch size follows a geometric series, so $q_{min,j}$ is given by

$$q_{min,j} = Q \times A_j \tag{3.1}$$

where,

$$A_j = \frac{\frac{Max(P)_{j,j+1}}{Min(P)_{j,j+1}} - 1}{\left(\frac{Max(P)_{j,j+1}}{Min(P)_{j,j+1}}\right)^{m_j} - 1}$$

And $q_{i,j}$ can be determined from;

$$q_{i,j} = q_{min,j}\left(\frac{Max[P_j, P_{j+1}]}{Min[P_j, P_{j+1}]}\right)^{i-1}, \ for\ i = 1, \cdots, m_j. \tag{3.2}$$

So the lot size will be their geometric sum. i.e.

$$Q = q_{min,j}\sum_{i=1}^{mj}\left(\frac{Max[P_j, P_{j+1}]}{Min[P_j, P_{j+1}]}\right)^{(i-1)} \tag{3.3}$$

The total cost will be the sum of inventory cost, setup and transportation cost; i.e. $C(Q, M) = IHC + STC$ where,

$$IHC = \sum_{j=1}^{n} h_j\left[\frac{Q^2 A_j}{Max(P)_{j,j+1}} + \frac{Q^2}{2}\left(\frac{1}{Min(P)_{j,j+1}} - \frac{1}{Max(P)_{j,j+1}}\right)\right] \tag{3.4}$$

and

$$STC = \sum_{j=1}^{n}[S_j + T_j m_j] \tag{3.5}$$

After simplifying equations (3.4)and (3.5) and rearranging terms, we have the following expression for total cost

$$C(Q,M) = Q\alpha D + \sum_{j=1}^{n} \frac{Q\gamma_j D}{[(\delta_j)^{m_j} - 1]} + \sum_{j=1}^{n} (S_j + T_j m_j)\frac{D}{Q}$$

Where,

$$\alpha = \sum_{j=1}^{n} \frac{1}{2}\left(\frac{1}{min(P)_{j,j+1}} - \frac{1}{max(P)_{j,j+1}}\right)$$

$$\gamma_j = \frac{h_j(\delta_j - 1)}{max(P)_{j,j+1}} \; for \; j = 1, \cdots, n$$

And

$$\delta_j = \frac{max(P)_{j,j+1}}{min(P)_{j,j+1}}$$

Decision variables are the number of batches, m_j, in a stage j, and lot size, Q.

3.2 Model and cost function after integrating Quality

3.2.1 Problem Definition

In this section it is assumed that the quality of the output of the various stages is not perfect. For each cycle, at the j^{th} level, the production process starts in the in control state and after some time it may shift at a random time to an out of control state and starts producing a fixed fraction α of nonconforming items. The restoration work is carried out after producing each batch if the process is found out to be in out of control state. Also the items are screened out before passing to the next production stage, and the inspection is error free. The time to shift distribution at stage j is assumed to be exponential with mean θ_j i.e.

$$\begin{aligned} f_j(t) &= \frac{1}{\theta_j}\, e^{\frac{-t}{\theta_j}} \\ &= \frac{1}{\theta_j}\, e^{\frac{-Q}{P_j\theta_j}} \end{aligned}$$

The restoration and inspection time is negligible and so they are neglected.

Assumptions

1. It is assumed that the processes are in the in control state at start of the production cycle, production items of acceptable quality. However, after some time the production process may shift to an out of control state.

2. The elapsed time t for which the process remain in the in control state, before the shift occurs is considered to be a random variable with known probability distribution.

3. Once in the out of control state the process starts producing a fixed percentage of defective items and stays in that state till either the end of production run or some restoration action.

4. In the presence of deteriorating production processes, nonconforming items must be screened before any shipment is made, so that they are not passed to subsequent stage to avoid unnecessary processing.

5. At stage j, the production process produces m_j batches. At the end of the

production of each batch, an inspection of the process is carried out. If the process is found in control, it continues producing the next batch. Otherwise, the process is restored to the in control state ant an additional cost which is assumed to be proportional to the detection delay, i.e. the time elapsed while the process is in the out of control state. The restoration time is negligible.

6. It is assumed that the screening of nonconforming items is error free.

7. The time taken by restoration and shipment is negligible and hence ignored.

Additional notations after integrating the *Quality*

Variables

Q_j = The lot size at stage j.

Q_{n+1} = Final lot size of the product.

$N_{i,j}$ = Number of non-conforming items of batch i at stage j.

$G_{i,j}$ = Size of the good items of batch i at stage j i.e.

= $G_{i,j} = q_{i,j} - N_{i,j}$.

Parameters

$t_{i,j}$ = Production time of batch i at stage j.

θ_j = Mean time to shift at stage j.

α = Percentage of nonconforming units produced when the process is in the *out of control* state.

s = Unit cost of producing a nonconforming item.

The expected total cost consists of setup costs, the inventory carrying costs, the quality related cost due to the production of nonconforming items and additional cost due to the restoration of the process to the in-control state, if it is found to be out of control state. These various costs are now derived as follows;

Inventory Holding Cost

The inventory carrying cost is affected by the number of nonconforming items produced at each stage. The number of non-conforming items produced in a batch i at stage j will be, $for\ i = 1, \cdots, m_j;\quad for\ j = 1, \cdots, n$

$$\begin{aligned} N_{i,j} &= \int_0^{t_{i,j}} \alpha P_j (t_{i,j} - t) f(t_{i,j}) dt \\ N_{i,j} &= \alpha P_j \left[\frac{q_{i,j}}{P_j} - \theta_j + \theta_j e^{\frac{-q_{i,j}}{P_j \theta_j}} \right] \ for\ i = 1, \cdots, m_j \end{aligned} \tag{3.6}$$

Nonconforming items at each level are removed and and not passed to the following stage. Hence the batch size transferred to the next stage i.e. $G_{i,j}$ can be obtained as follows for $j = 1, \cdots, n$;

$$G_{i,j} = q_{i,j} - N_{i,j},\ for\ i = 1, \cdots, m_j \tag{3.7}$$

Where $q_{i,j}$ is determined from equation (3.2) and $N_{i,j}$ is from equation (3.6).

Considering Fig. 3.3, it is clearly visible that the lot size which the succeeding stage received will be the difference of the nonconforming items produced by the preceding stage from the batch size produced by that stage, therefore the size of the lot shifted to the next stage calculated recursively as,

$$Q_{j+1} = Q_j - \sum_{i=1}^{mj} N_{i,j}, \; for\ j = 1, \cdots, n \tag{3.8}$$

Considering Figure 3.4 after integrating the *quality*, which corresponds to the case where three batches are produced at stage j, i.e. $m_j = 3$ and where $P_j < P_{j+1}$. The inventory area can be described by the areas $E_1, \ldots, E_{10}$ as follows, for $j = 1, \cdots, n$,

$$\begin{array}{rcl}
E_1 = \frac{q_{3,j}^2}{2P_j} & , & E_2 = \frac{P_{j+1}-P_j}{2P_{j+1}^2}[G_{3,j}]^2 \\
E_3 = \frac{P_j}{P_{j+1}^2}[G_{3,j}]^2 & , & E_4 = \frac{P_j}{P_{j+1}^2}[N_{3,j} \times G_{3,j}] \\
E_5 = \frac{P_j}{2P_{j+1}^2}[N_{3,j}]^2 & , & E_6 = \frac{P_j}{2P_{j+1}^2}[N_{2,j}]^2 \\
E_7 = \frac{P_{j+1}-P_j}{2P_{j+1}^2}[G_{2,j}]^2 & , & E_8 = \frac{G_{1,j}^2}{2P_{j+1}} \\
E_9 = \frac{P_j}{P_{j+1}^2}[G_{2,j}]^2 & , & E_{10} = \frac{P_j}{P_{j+1}^2}[N_{2,j} \times G_{2,j}]
\end{array}$$

These ten areas can be summarized in five areas which can be generalized to any value of the number of batches m_j at stage j.

Let

$$\begin{array}{rcccl}
A_{1,j} & = & E_1 + E_8 & = & \frac{q_{3,j}^2}{2P_j} + \frac{G_{1,j}^2}{2P_{j+1}} \\
A_{2,j} & = & E_2 + E_7 & = & \frac{P_{j+1}-P_j}{2P_{j+1}^2} \sum_{i=2}^{3} \left[G_{i,j}\right]^2 \\
A_{3,j} & = & E_3 + E_9 & = & \frac{P_j}{P_{j+1}^2} \sum_{i=2}^{3} \left[G_{i,j}\right]^2 \\
A_{4,j} & = & E_4 + E_10 & = & \frac{P_j}{P_{j+1}^2} \sum_{i=2}^{3} \left[N_{i,j} G_{i,j}\right] \\
A_{5,j} & = & E_5 + E_6 & = & \frac{P_j}{2P_{j+1}^2} \sum_{i=2}^{3} \left[N_{i,j}\right]^2
\end{array}$$

This can be generalized for any number of batches, m_j as follows;

For $m_j = 1$

$$A_{1,j} = \frac{q_{1,j}^2}{2P_j} + \frac{G_{1,j}^2}{2P_{j+1}} \tag{3.9}$$

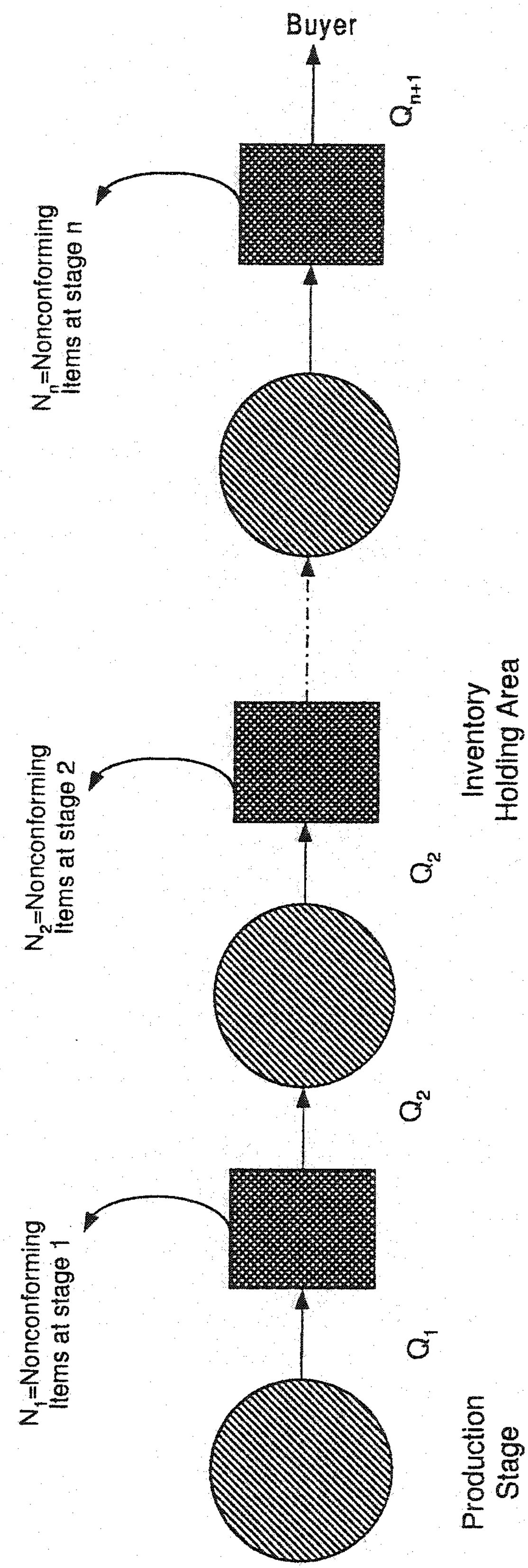

Figure 3.3: Inventory flow diagram in MS-PIS, after integrating the Quality

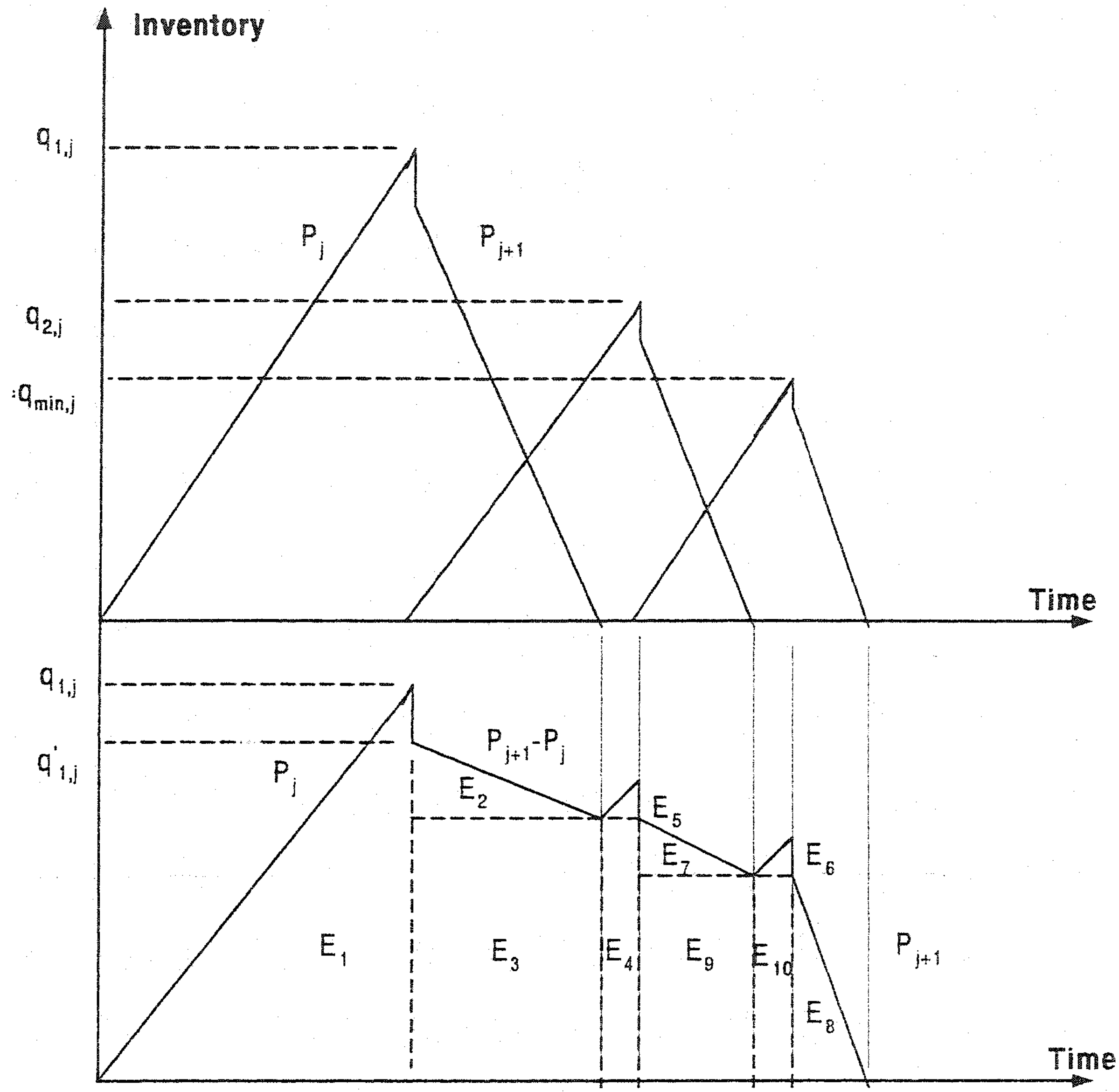

Figure 3.4: Inventory profile after integrating the Quality and Restoration Aspects (when $P_j < P_{j+1}$)

and for $m_j = 2, 3, \cdots$

$$
\begin{aligned}
A_{1,j} &= \frac{q_{m_j,j}}{2P_j} + \frac{G_{1,j}^2}{2P_{j+1}} \\
A_{2,j} &= \frac{P_{j+1} - P_j}{2P_{j+1}^2} \sum_{i=2}^{m_j} \left[G_{i,j}\right]^2 \\
A_{3,j} &= \frac{P_j}{P_{j+1}^2} \sum_{i=2}^{m_j} \left[G_{i,j}\right]^2 \\
A_{4,j} &= \frac{P_j}{P_{j+1}^2} \sum_{i=2}^{m_j} \left[N_{i,j} G_{i,j}\right] \\
A_{5,j} &= \frac{P_j}{2P_{j+1}^2} \sum_{i=2}^{m_j} \left[N_{i,j}\right]^2 \qquad (3.10)
\end{aligned}
$$

The inventory area at stage j is simply;

$$
\begin{aligned}
A_j &= A_{1,j} && \text{for } m_j = 1 \\
&= \sum_{k=1}^{5} A_{k,j} && \text{for } m_j \geq 2
\end{aligned}
$$

Similarly, From figure 3.5, for $m_j = 3$ and $P_j > P_{j+1}$, the inventory area will again be the sum of the areas $E_1, \ldots, E_{10}$;

$$
\begin{aligned}
E_1 &= \tfrac{q_{1,j}^2}{2P_j} &&, \quad E_2 = \tfrac{P_j - P_{j+1}}{2P_{j+1}^2}[G_{1,j}]^2 \\
E_3 &= \tfrac{[G_{1,j}]^2}{P_{j+1}} &&, \quad E_4 = \tfrac{P_j}{P_{j+1}^2}[N_{1,j} G_{1,j}] \\
E_5 &= \tfrac{P_j}{2P_{j+1}^2}[N_{1,j}]^2 &&, \quad E_6 = \tfrac{P_j}{2P_{j+1}^2}[N_{2,j}]^2 \\
E_7 &= \tfrac{P_j - P_{j+1}}{2P_{j+1}^2}[G_{2,j}]^2 &&, \quad E_8 = \tfrac{G_{3,j}^2}{2P_{j+1}} \\
E_9 &= \tfrac{[G_{2,j}]^2}{P_{j+1}} &&, \quad E_{10} = \tfrac{P_j}{P_{j+1}^2}[N_{2,j} \times G_{2,j}]
\end{aligned}
$$

This can be generalized as follows for $j = 1, \cdots, n$ and for $m_j = 1$,

$$
A_{1,j} = \frac{q_{1,j}^2}{2P_j} + \frac{G_{1,j}^2}{2P_{j+1}} \qquad (3.11)
$$

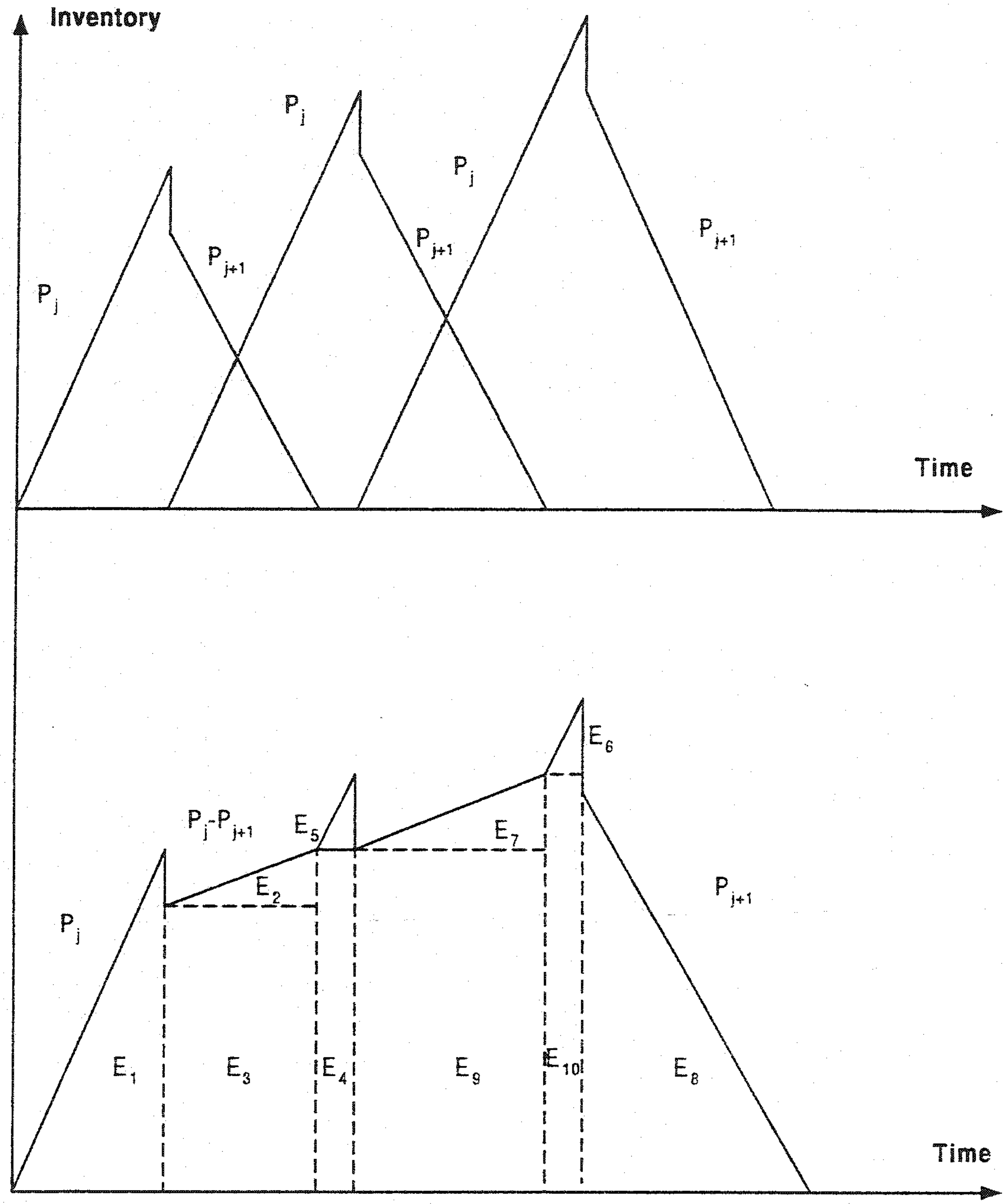

Figure 3.5: Inventory profile after integrating the Quality and Restoration Aspects (when $P_j > P_{j+1}$)

and for any value of $m_j \geq 2$,

$$
\begin{aligned}
A_{1,j} &= \frac{q_{1,j}^2}{2P_j} + \frac{G_{m_j,j}^2}{2P_{j+1}} \\
A_{2,j} &= \frac{P_j - P_{j+1}}{2P_{j+1}^2} \sum_{i=1}^{m_j-1} [G_{i,j}]^2 \\
A_{3,j} &= \frac{\sum_{i=1}^{m_j-1} [G_{i,j}]^2}{P_{j+1}} \\
A_{4,j} &= \frac{P_j}{P_{j+1}^2} \sum_{i=1}^{m_j-1} [N_{i,j} G_{i,j}] \\
A_{5,j} &= \frac{P_j}{2P_{j+1}^2} \sum_{i=1}^{m_j-1} [N_{i,j}]^2
\end{aligned}
\tag{3.12}
$$

The expected inventory holding cost per unit time is then given by;

$$EHC = \frac{D}{Q_{n+1}} \sum_{j=1}^{n} [h_j A_j] \tag{3.13}$$

Setup and Transportation Cost

The setup and transportation cost per unit time for a j-stage system can be determined from equation (3.5) and is given by;

$$STC = \frac{D}{Q_{n+1}} \sum_{j=1}^{n} [S_j + T_j \; m_j] \tag{3.14}$$

Quality Cost

Using equation (3.6), the total number of nonconforming items produced in the cycle will be;

$$\sum_{j=1}^{n} \sum_{i=1}^{m_j} N_{i,j} = \sum_{j=1}^{n} \sum_{i=1}^{m_j} \alpha P_j \left[\frac{q_{i,j}}{P_j} - \theta + \theta e^{\frac{-q_{i,j}}{P_j \theta_j}} \right]$$

Hence the expected quality related cost per cycle is given by;

$$EQC = \frac{sD}{Q_{n+1}} \sum_{j=1}^{n} \sum_{i=1}^{m_j} N_{i,j} \tag{3.15}$$

Restoration Cost

The restoration cost is assumed to be a linear function of the detection delay, i.e.

$$R(t_j - t) = r_0 + r_1(t_j - t)$$

where, r_0 and r_1 are restoration cost parameters and are constant. Detection delay is defined to be the time elapsed from the time the shift to the out of control state occurs until the time it is detected in the following inspection. Since restoration is performed after the completion of each batch, so the expected restoration cost during the i^{th} batch of the process at stage j will be calculated as follows;

$$\begin{aligned} ERC_{i,j} &= \int_0^{t_{i,j}} \frac{1}{\theta_j}(r_{0,j} + r_{1,j}(t_{i,j} - t))e^{-t/\theta_j} \\ &= \left[r_{0,j} + r_{1,j}\frac{q_{i,j}}{P_j} - r_{1,j}\theta_j + (r_{1,j}\theta_j - r_{0,j})e^{\frac{q_{i,j}}{P_j\theta_j}} \right] \end{aligned} \tag{3.16}$$

So the expected restoration cost for the whole system will be;

$$ERC = \frac{D}{Q_{n+1}} \sum_{j=1}^{n} \sum_{i=1}^{mj} \left[(r_{0,j} - r_{1,j}\theta_j) + r_{1,j}\frac{q_{i,j}}{P_j} + (r_{1,j}\theta_j - r_{0,j})exp\left(-\frac{q_{i,j}}{P_j\theta_j}\right) \right] \tag{3.17}$$

This completes the derivation of all cost components.

3.2.2 The Total Cost

The expected total cost per unit time is simply the sum of equations (3.13), (3.15), (3.14) and (3.17), that is

$$ETC = EHC + EQC + STC + ERC$$

The problem is to determine the lot size and number of batches and consequently the inspection schedule at each stage such that the expected total cost per unit time is minimized.

Numerical Example

In this section, we illustrate the model using a three stage production system. The pattern search technique of Hooke and Jeeves [60] was used to minimize the cost function. This algorithm was coded and run on a personal computer. The example data is summarized in Table 3.1. Same data had been employed that was used in [1]. Additional cost parameters were added to take into consideration the processes shift and quality related data.

j	P_j	h_j	S_j	T_j	s	$r_{0,j}$	$r_{1,j}$
1	2040	0.5	10	1	5	1.00	0.150
2	5000	0.5	18.3	9	5	1.83	0.275
3	4000	0.04	15	5	5	1.50	0.225

Table 3.1: Problem Parameters

The Method of Hooke and Jeeves

The method of Hooke and Jeeves performs two types of search-exploratory search and pattern search. The first two iterations of the procedure are illustrated in Figure 3.6. Given x_1, an exploratory search along the coordinate directions produces the point x_2. Now a pattern search along the direction $x_2 - x_1$ leads to the point y. Another exploratory search starting from y gives the point x_3. The next pattern search is along the direction $x_3 - x_2$, yielding y'. The process is then repeated.

Initialization step Let $d_1, \cdots, d_n$ be the coordinate directions. Choose a scalar $\varepsilon > 0$ to be used for terminating the algorithm. Furthermore, choose an initial step size, $\Delta \geq \varepsilon$, and an acceleration factor, $\alpha > 0$. Choose a starting point x_1, let $y_1 = x_1$, let $k = j = 1$, and go to main step.

Main Step

1. If $f(y_j + \Delta d_j) < f(y_j)$, the trial is termed a success; let $y_{j+1} = y_j + \Delta d_j$, and go to step 2. If, however, $f(y_j + \Delta d_j) \geq f(y_j)$, the trial is deemed a failure. In this case, if $f(y_j + \Delta d_j) < f(y_j)$, let $y_{j+1} = y_j - \Delta d_j$, and go to step 2; if $f(y_j + \Delta d_j) \geq f(y_j)$, let $y_{j+1} = y_j$, and go to step 2.

2. If $j < n$, replace j by $j + 1$, and repeat step 1. Otherwise, go to step 3, if $f(y_{n+1}) < f(x_k)$, and go to step 5 if $f(y_{n+1}) \geq f(x_k)$.

3. Let $x_{k+1} = y_{n+1}$, and lt $y_1 = x_{k+1} + \alpha(x_{k+1})$. Replace k by $k+1$, let j=1, and go to step 1.

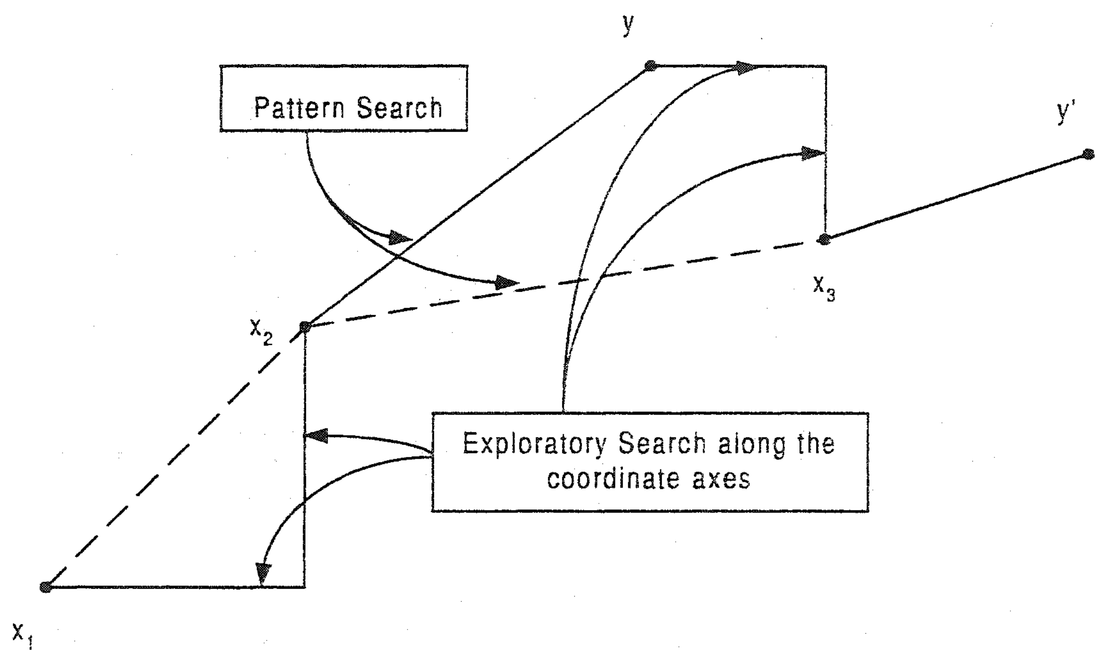

Figure 3.6: Illustration of the method of Hooke and Jeeves.

4. if $\Delta < \varepsilon$, stop; x_k is the solution. Otherwise, replace Δ by $\Delta/2$. Let $y_1 = x_k$, $x_{k+1} = x_k$, replace k by $k+1$, let $j = 1$, and repeat step 1.

It should be noted that step 1 and 2 above describe an exploratory search. Furthermore, step 3 is an acceleration step along the direction $x_{k+1} - x_k$. Note that a decision whether to accept or reject the acceleration step is not made until after an exploratory search is performed. In step 4, the step size Δ is reduced.

3.2.3 Conclusion

Solution to the perfect lot sizing model calculated in [1] is summarized in Table 3.2. Results are reported for the case of the same number of batches at each stage as well as for the general case of having different batches.

For Same Number of Batches

m_j	Q	$Cost$
3	801.87	220.24

For Different Numbers of Batches

m_1	m_2	m_3	Q	$Cost$
5	3	1	762.99	210.49

Table 3.2: Results for Perfect Production Processes.

Numerical results are presented in Table 3.3 and 3.4 to illustrate the effect of model developed in section 3.2 on the original one, for a range of α and θ, different parameters are computed to see their effect. The optimum values of lot sizes at different stages, the number of batches in each stage, and the quality, restoration and total expected costs are obtained. Table 3.3 and 3.4 summarize the results obtained for the case of identical and different number of batches through all stages respectively. As in [1], considering different batches at the various stages yields lower cost then the case of equal batches because of the difference in transportation costs. The possibility of process shift to an out of control stage and presence of nonconforming items result in larger lot size at initial stages. Compared to the perfect case, smaller batch sizes are produced due to the risk of producing nonconforming items. Considering Figure 3.7, quality related costs will be higher when producing high proportion of nonconforming items, because of this, the total cost is more when having high nonconforming items, see Figure 3.8. So it can be concluded that the lot size and expected total cost are sensitive to the proportion of defective items.

For $\alpha = 0.01$

θ	m_j	Q_1	Q_2	Q_3	Q_4	QC	RC	$Cost$
0.01	3	790.37	783.02	776.68	769.94	132.30	16.08	372.63
0.05	3	695.53	690.90	688.46	684.70	78.72	11.67	315.32
0.1	3	695.68	692.29	690.88	688.34	52.96	7.93	284.98
0.5	3	753.52	752.30	751.92	751.12	15.63	2.21	238.95
1.0	3	773.62	772.91	772.71	772.27	8.430	1.17	230.21

For $\alpha = 0.05$

θ	m_j	Q_1	Q_2	Q_3	Q_4	QC	RC	$Cost$
0.01	4	557.95	533.69	516.16	495.37	631.16	28.52	939.25
0.05	3	382.44	372.73	368.69	361.60	287.37	15.80	600.73
0.1	3	434.08	426.26	423.43	417.96	192.17	9.36	473.00
0.5	3	611.89	607.88	606.64	603.99	64.94	2.27	297.69
1.0	3	682.63	680.00	679.21	677.50	37.50	1.18	262.91

For $\alpha = 0.1$

θ	m_j	Q_1	Q_2	Q_3	Q_4	QC	RC	$Cost$
0.01	4	269.31	248.70	237.79	221.91	1066.94	50.52	1618.17
0.05	3	250.11	240.23	236.67	230.02	435.68	18.57	873.28
0.1	3	308.75	300.06	297.17	291.41	296.65	10.25	652.95
0.5	3	509.05	503.42	501.72	498.05	109.96	2.32	359.83
1.0	3	599.59	595.52	594.32	591.68	66.41	1.19	299.42

For $\alpha = 0.25$

θ	m_j	Q_1	Q_2	Q_3	Q_4	QC	RC	$Cost$
0.01	3	81.80	71.07	67.37	61.27	1671.48	91.19	3216.18
0.05	3	140.35	131.18	128.39	122.90	708.15	21.95	1468.55
0.1	3	187.11	178.30	175.68	170.29	492.16	11.39	1048.38
0.5	3	361.69	354.41	352.29	347.68	200.72	2.39	507.21
1.0	3	462.11	456.02	454.25	450.36	129.63	1.21	390.05

Table 3.3: Results for imperfect Production Processes and Equal number of batches

For $\alpha = 0.01$

θ	m_1	m_2	m_3	Q_1	Q_2	Q_3	Q_4	QC	RC	$Cost$
0.01	5	3	2	797.08	789.88	783.47	776.42	132.37	16.43	363.12
0.05	5	3	2	695.75	691.44	689.00	684.98	78.31	11.84	305.48
0.1	5	3	2	696.03	692.91	691.50	688.75	52.47	8.01	275.02
0.5	5	3	2	758.25	757.13	756.75	755.87	15.44	2.22	229.10
1.0	5	3	2	779.61	778.96	778.76	778.27	8.320	1.17	220.43

For $\alpha = 0.05$

θ	m_1	m_2	m_3	Q_1	Q_2	Q_3	Q_4	QC	RC	$Cost$
0.01	5	4	2	517.81	495.86	480.07	459.88	629.27	29.16	928.94
0.05	5	3	2	376.69	367.96	364.01	356.53	281.94	16.08	586.95
0.1	5	2	2	380.53	374.83	371.69	366.99	183.77	9.55	458.05
0.5	5	2	2	548.76	545.81	544.36	542.01	61.78	2.28	286.33
1.0	5	3	2	684.48	682.10	681.31	679.42	36.84	1.18	252.65

For $\alpha = 0.1$

θ	m_1	m_2	m_3	Q_1	Q_2	Q_3	Q_4	QC	RC	$Cost$
0.01	5	4	3	257.03	237.93	227.79	212.56	1045.01	52.24	1597.32
0.05	5	2	2	217.66	210.51	206.60	200.92	415.34	19.02	846.94
0.1	5	2	2	272.81	266.48	263.19	258.17	282.46	10.42	629.31
0.5	5	2	2	456.61	452.50	450.49	447.23	104.37	2.33	345.23
1.0	5	2	2	540.01	537.02	535.59	533.23	63.05	1.19	287.53

For $\alpha = 0.25$

θ	m_1	m_2	m_3	Q_1	Q_2	Q_3	Q_4	QC	RC	$Cost$
0.01	5	2	2	69.24	61.60	57.66	52.60	1577.70	94.80	3134.17
0.05	5	2	2	124.65	117.93	114.68	109.82	672.96	22.26	1408.97
0.1	5	2	2	167.33	160.87	157.77	152.97	467.71	11.50	1003.25
0.5	5	2	2	326.26	320.91	318.36	314.22	190.80	2.40	484.72
1.0	5	2	2	416.77	412.30	410.18	406.70	123.17	1.21	374.03

Table 3.4: Results for imperfect Production Processes and Different Number of Batches

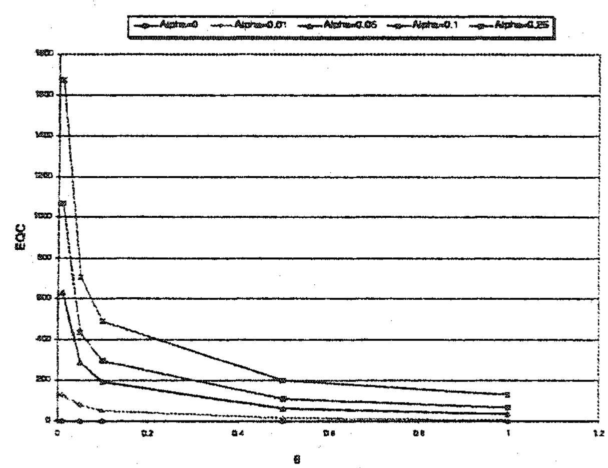

Figure 3.7: Graph between θ and EQC, for same number of batches

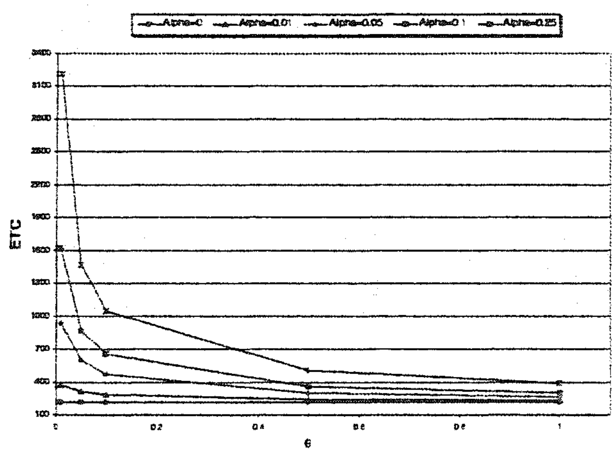

Figure 3.8: Graph between θ and ETC, for same number of batches

Chapter 4

Integrated vendor buyer Relationship (with Deterministic demand)

The purpose of this chapter is to integrate quality, restoration and maintenance aspects to a joint economic lot size model under deterministic condition. The joint economic lot size ($JELS$) models consists of formulas based on the joint optimization of manufacturer (vendor) and purchaser (buyer) costs, where the objective is to minimize the total inventory related costs of vendor and buyer.

4.1 Overview

Inventory is usually classified as one of the current assets of an organization. The control of inventories of physical goods is a problem common to all enterprises. A small decrease in inventory cost could result in an unprecedented increase in the profit of an organization. Therefore the companies are allocating an increasing proportion of resources to control the inventory.

In the vendor-buyer integrated model the problem facing the buyer is how much to order to minimize its inventory as well as shipment cost. On the other hand, the problem facing the vendor is to determine the economic batch quantity and shipment size that will be supplied to the buyer. Several researchers have shown that in integrated models one partner's gain exceeds the other partner's loss, thus the net benefit can be shared by both parties (Buyer and the Vendor).

The adoption of integrated models (Buyer and vendor) can contribute significantly to a better buyer vendor problem, specially its importance has been heightened since an increasing number of organizations are implementing just in time (JIT), where the basic objective is to purchase and manufacture the required items just in time for their consumption. The successful implementation of JIT requires a new spirit of cooperation between the buyer and the vendor. Starting with the shipment pattern tackled by Goyal [2] a model has been developed after incorporating the imperfect production process and restoration aspect.

4.1.1 Basic Model Description

The problem considered by Goyal [2] is that of a single vendor (manufacturer) supplying a single buyer with a product. The vendor manufactures, at a finite rate, in n batches and incurs a batch setup cost. Each batch is dispatched to the buyer in n number of shipments and the vendor (and /or buyer) incurs fixed order/delivery cost associated with each shipment. The first shipment will be of small size i.e., q_1, followed by $(n-1)$ equal sized shipment of size,

$$\begin{aligned} &= \text{first shipment size} \times \frac{\text{Rate of production}}{\text{Demand Rate}} \\ &= q_1 \frac{P}{D} \\ &= q_2 \end{aligned}$$

The inventory profile for vendor and buyer is depicted in Figure 4.1. The total stock in the system is at a minimum when the production of a batch is just about to start. At this point the vendor stock is zero and the buyer's stock is just enough to satisfy the demand until the first shipment of the next batch arrives, the size of this safety stock will be $\frac{q_1 D}{P}$. The total cost will be sum of inventory holding cost and the setup and transportation cost, and the decision variables are the first batch size i.e., q_1 and the number of batches (n).

Assumptions For the Perfect Process Model

1. The production rate is always greater then the demand rate.
2. Demand for the item is constant over time.
3. Shortages are not allowed.
4. Manufacturing setup cost, unit inventory cost for the vendor and the buyer and the cost of a shipment from the vendor to the buyer are known.
5. Minimization of cost is the objective.
6. The transportation time of the batches to the buyer is neglected.

7. The inventory holding cost of buyer is always greater then that of the vendor.

Notation

In deriving the cost model the following notations will be used.

q_1 = Size of the first shipment.

n = Number of Batches transported to the Buyer.

P = Production rate of the vendor.

D = Demand rate of the Buyer.

λ = Ratio between Production and Demand rate.

q_2 = Size of the successive batches produced by the vendor, $= q_1\lambda$.

Q = The lot size.

$= q_1 + (n-1)q_2$.

h_v = Stock holding cost per unit per year for the vendor.

h_b = Stock holding cost per unit per year for the buyer.

A_v = Cost of a production setup.

A_b = Cost of a shipment from the vendor to the buyer.

SSC = Production setup and shipment Cost.

IHC = Joint inventory holding cost for both vendor and buyer.

TC = The total annual cost of the integrated (vendor-buyer) system.

= Setup and shipment cost + Inventory holding cost for the vendor and buyer.

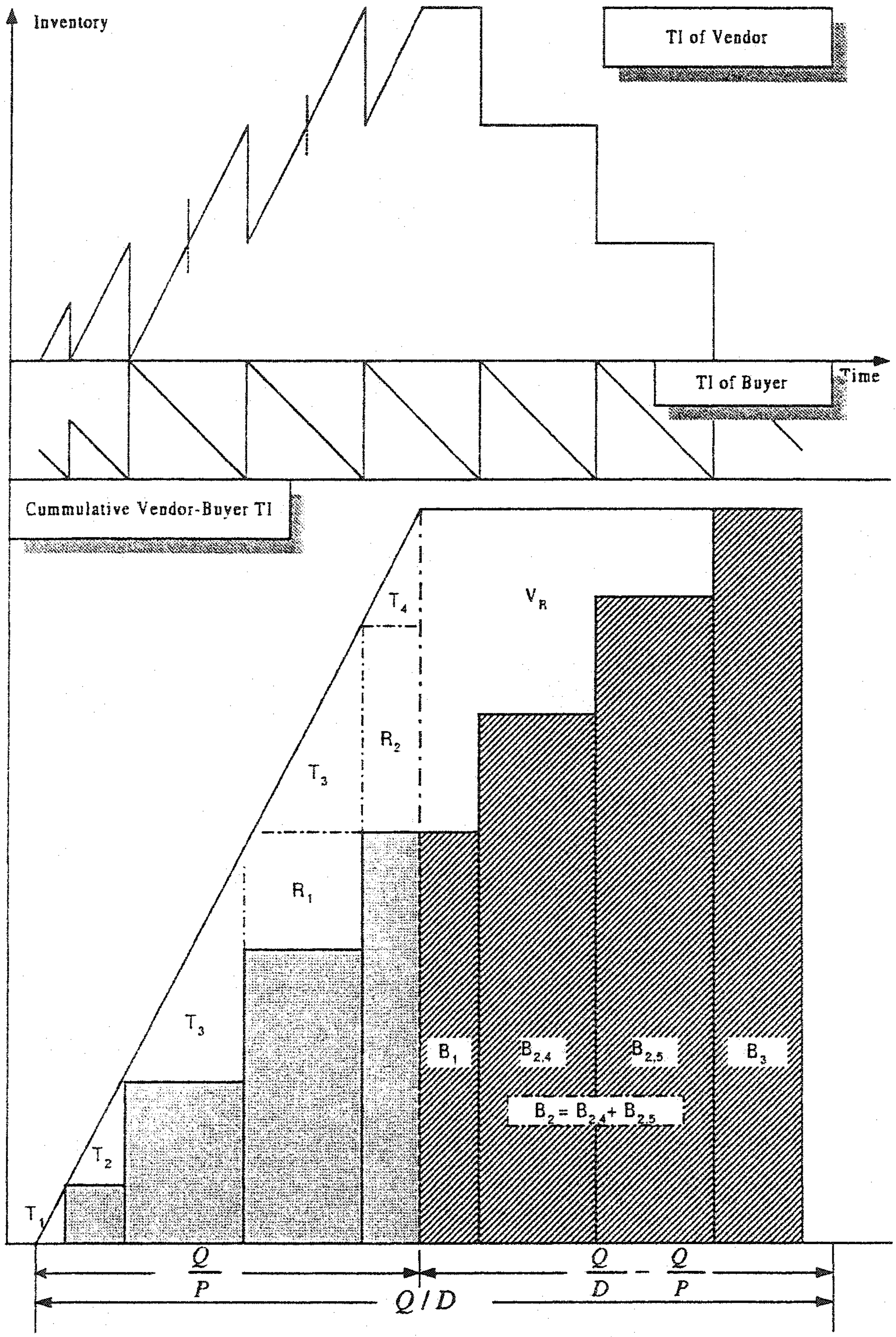

Figure 4.1: The inventory profile of combined vendor buyer system with perfect production process (when $n = 7$).

The total annual cost of the vendor and buyer is given by equation (4.1).

$$\begin{aligned} TC(n, q_1) &= \frac{(A_v + nA_b)D}{q_1(1+(n-1)\lambda)} + \frac{q_1}{2}\left[h_v \frac{2D + (P-D)(1+(n-1)\lambda)}{P}\right] \\ &+ \frac{q_1(h_b - h_v)}{2}\left[\frac{(1+(n-1)\lambda^2)}{(1+(n-1)\lambda)}\right] \end{aligned} \tag{4.1}$$

The problem is to find the size of the first shipment q_1 and the number of shipments that minimizes the total cost for the system.

Total Inventory (TI) of Vendor

Considering Figure 4.1, the total inventory of vendor can be also be found out by subtracting the cumulative buyer TI from the cumulative vendor TI. Let $'i'$ be the smallest integer such that;

$$i\left(\frac{q_2}{D}\right) + \frac{q_1}{D} + \frac{q_1}{P} \geq \frac{Q}{P}$$

The total area will consists of following triangles and rectangles.

$$\begin{aligned} T_1 &= \frac{(q_1)^2}{2P} \\ T_2 &= \frac{(q_2)^2}{2P} \\ T_3 &= (i-1)\frac{P}{2}\left(\frac{q_2}{D}\right)^2 \\ T_4 &= \frac{P}{2}\left[\frac{Q}{P} - \left[(i-1)\left(\frac{q_2}{D}\right) + \frac{q_1}{D} + \frac{q_1}{P}\right]\right]^2 \\ R_1 &= \sum_{j=1}^{i-2} R_j \quad \text{if } i \geq 3 \end{aligned}$$

where,

$$R_j = \left[\frac{q_2}{D}\right] j \left[P\frac{q_2}{D} - q_2\right] \quad \text{for} j = 1, \cdots, i-2$$

$$
\begin{aligned}
R_2 &= (i-1)\left[P\frac{q_2}{D}-q_2\right]\left(\frac{Q}{P}-\left[(i-1)\left(\frac{q_2}{D}\right)+\frac{q_1}{D}+\frac{q_1}{P}\right]\right) \\
V_R &= Q\left[\frac{Q}{D}-\frac{Q}{P}\right] \\
B_1 &= (q_1+iq_2)\left[i\frac{q_2}{D}+\frac{q_1}{D}+\frac{q_1}{P}-\frac{Q}{P}\right]
\end{aligned}
$$

if $n-i=3$ then

$$B_2 = (q_1+(n-2)q_2)\frac{q_2}{D} \tag{4.2}$$

if $n-i>3$ then

$$B_2 = \sum_{j=i+1}^{n-2} B_{2,j} \tag{4.3}$$

where,

$$B_{2,j} = (q_1+jq_2)\frac{q_2}{D} \quad \text{for} j = i+1,\cdots,n-2$$

and

$$B_3 = Q\left[\frac{q_2}{D}-\frac{q_1}{P}\right]$$

We can now write the TI expression for vendor as;

$$TI_{vendor} = \sum_{l=1}^{4} T_l + \sum_{m=1}^{2} R_m + \left(V_R - \sum_{n=1}^{3} B_n\right) \tag{4.4}$$

4.2 Integrated Imperfect Process Model

In this section, it is assumed that the process starts in the in control state and after some time shifts to an out of control state where it starts producing some nonconforming items, the time to shift is assumed to follow an exponential distribution with mean θ, the process is restored after producing each batch if it is found in the out of control state, and the nonconforming items are screened out right before a shipment is made.

4.2.1 Problem Definition

All earlier models addressing the vendor buyer problem considered perfect production processes. However, this may not be the case in many practical situation. Here, it is assumed that the process starts in the in-control state and then shifts to an out of control state at a random point in time with a known probability distribution function. There, it starts producing a fraction of nonconforming items, until it is restored and brought back to the in control state. The restoration work is performed after completion of batches if the process is found in the out of control state. The nonconforming items are screened out before shipping to the buyer and whenever the production stops. The total cost will be the sum of the inventory holding cost, setup and shipment cost, the restoration cost and quality cost due to producing nonconforming items. The problem is to find the size of the first shipment and the number of shipment that minimizes the total expected cost per unit time.

Notation

The following additional notations will be used to develop the model:

N_j = Number of non conforming items produced in batch j.

G_1 = The first shipment size received by the buyer, $= q_1 - N_1$.

$q_2, \cdots, q_n$ = Size of the successive batches produced by the vendor (other than the first one). $= G_1\lambda$.

$G_2, \cdots, G_n$ = Size of the shipment which the buyer received (other then the first one), $q_j - N_j$; $(j = 2, \cdots, n)$.

Q_v = The lot size produced by the vendor.

Q_b = The lot size received by the buyer.

EQC = Expected Quality Cost.

ERC = Expected Restoration Cost.

Assumptions For the Imperfect Process Model

In addition to the assumptions made for the perfect model, the following assumptions will be used for developing the integrated imperfect model;

1. The process starts in the in control state producing items of good quantity and then shifts to an out of control state where it starts producing a fraction of nonconforming items.

2. The process is restored after producing each batch if it is found in the out of control state.

3. Product inspection is done before shipping it to the buyer.

4. The inspection of nonconforming items is error free.

5. The time taken by inspection and restoration is negligible.

6. The nonconforming items produced in any interval are proportional to the length of that interval.

The size of the first batch produced by the vendor will be such that it last until the buyer receives the second shipment:

$$\frac{G_1}{D} = \frac{q_2}{P}$$

The safety stock which the buyer has at the start of the cycle is $\frac{q_1 D}{P}$.

Expected Quality Cost

In order to determine the expected quality cost, we need to find the number of nonconforming items produced by the vendor once the process shifts to an out of control state, therefore, it is assumed that the production process may shift at a random time to the out of control state and start producing a fixed fraction α of nonconforming items. It is assumed that this process shift follow an exponential distribution with mean θ. Hence the number of nonconforming items produced in batch j is;

$$N_j = \int_0^{t_j} \alpha P(t_j - t) f_j(t) dt \quad for j = 1, \cdots, n \tag{4.5}$$

where, $f_j(t) = \frac{1}{\theta} e^{\frac{-t}{\theta}}$ and t_j is the production time of batch j.

Let k represent the ratio between the consumption time of batch of size G_2 and the production time of batch of size q_2, i.e. the number of batches produced during the consumption of batch G_2;

$$k = \frac{G_2/D}{q_2/P}$$

From eq (4.5) we can determine the number of nonconforming items produced during different intervals.

$$
\begin{aligned}
N_1 &= \alpha P\left[\frac{q_1}{P} - \theta + \theta exp\left(-\frac{q_1}{P\theta}\right)\right] && (4.6) \\
N_j &= \alpha P\left[\frac{q_2}{P} - \theta + \theta exp\left(-\frac{q_2}{P\theta}\right)\right] \quad \text{for } j = 2, \cdots, n && (4.7) \\
\eta_k &= k\left[\alpha P\left(\frac{q_2}{P} - \theta + \theta exp\left(-\frac{q_2}{P\theta}\right)\right)\right] && (4.8)
\end{aligned}
$$

The product inspection policy calls for screening nonconforming items prior to each shipment. At the time of shipment, production of a batch might be underway. Part of the batch is produced before the shipment and the second part after the shipment. The nonconforming of this batch which are produced before the shipment will be screened at the time of shipment. The second part of the batch will be screened at the time of the next shipment. For simplicity we assume that the number of nonconforming items produced in each part is proportional to the corresponding production time. Hence, η_k represents the number of nonconforming items produced during interval G_2/D.

The lot size produced by the vendor will be;

$$Q_v = q_1 + (n-1)G_1\lambda \tag{4.9}$$

The lot size that the buyer receive (Q_b) will be the difference between the lot size produced by the vendor (Q_v) and the number of nonconforming items produced in all batches.

$$Q_b = Q_v - \sum_{j=1}^{n} N_j \tag{4.10}$$

Hence the expected quality cost per unit time due to producing nonconforming items is given by:

$$EQC = \frac{D}{Q_b} s \sum_{j=1}^{n} N_j \tag{4.11}$$

where, s is the unit cost of producing a nonconforming item by the vendor.

Inventory Holding Cost

Total Inventory of Buyer

Considering Figure 4.2, the total inventory of the buyer will simply be the sum of the areas of all shipments, i.e.

$$TI_{Buyer} = \frac{q_1^2}{2D} + \frac{(n-1)(G_2)^2}{2D} \tag{4.12}$$

Total Inventory of Vendor

The total inventory for the vendor can be obtained by breaking the areas into two parts; the area of first part will be the sum of areas, T_1, T_2, T_3, T_4, R_1 *and* R_2, which are calculated as follows, considering Figure (4.2), let i be the smallest integer such that,

$$i\left(\frac{G_2}{D}\right) + \frac{G_1}{D} + \frac{q_1}{P} \geq \frac{Q_v}{P}$$

Let t be the time interval such that,

$$t = \frac{Q_v}{P} - \left[(i-1)\left(\frac{G_2}{D}\right) + \frac{G_1}{D} + \frac{q_1}{P}\right]$$

Now, the corresponding areas can be find out as follows,

$$
\begin{aligned}
T_1 &= \frac{q_1^2}{2P} \\
T_2 &= \frac{(q_2)^2}{2P} \\
T_3 &= (i-1)\frac{P}{2}\left(\frac{G_2}{D}\right)^2 \\
T_4 &= \frac{P}{2}[t]^2 \\
R_1 &= \sum_{j=1}^{i-2} R_j \quad \text{if } i \geq 3
\end{aligned}
$$

where,

$$
\begin{aligned}
R_j &= \left[\frac{G_2}{D}\right] j \left[P\frac{G_2}{D} - \eta_k - G_2\right] \quad (\text{for} j = 1, \cdots, i-2) \\
R_2 &= (i-1)\left[P\frac{G_2}{D} - \eta_k - G_2\right](t) \qquad (4.13)
\end{aligned}
$$

The area of second part will be the difference between V_R and B_1, B_2 & B_3.

$$
\begin{aligned}
V_R &= Q_b\left[\frac{Q_b}{D} - \frac{Q_v}{P}\right] \\
B_1 &= (G_1 + iG_2)\left[i\frac{G_2}{D} + \frac{G_1}{D} + \frac{q_1}{P} - \frac{Q_v}{P}\right]
\end{aligned}
$$

if $n - i = 3$ then

$$
B_2 = (G_1 + (n-2)G_2)\frac{G_2}{D}
$$

if $n - i > 3$ then

$$
B_2 = \sum_{j=i+1}^{n-2} B_{2,j}
$$

where,

$$
\begin{aligned}
B_{2,j} &= (G_1 + jG_2)\frac{G_2}{D} \quad (\text{for} j = i+1, \cdots, n-2) \\
B_3 &= Q_b\left[\frac{G_2}{D} - \frac{q_1}{P}\right] \qquad (4.14)
\end{aligned}
$$

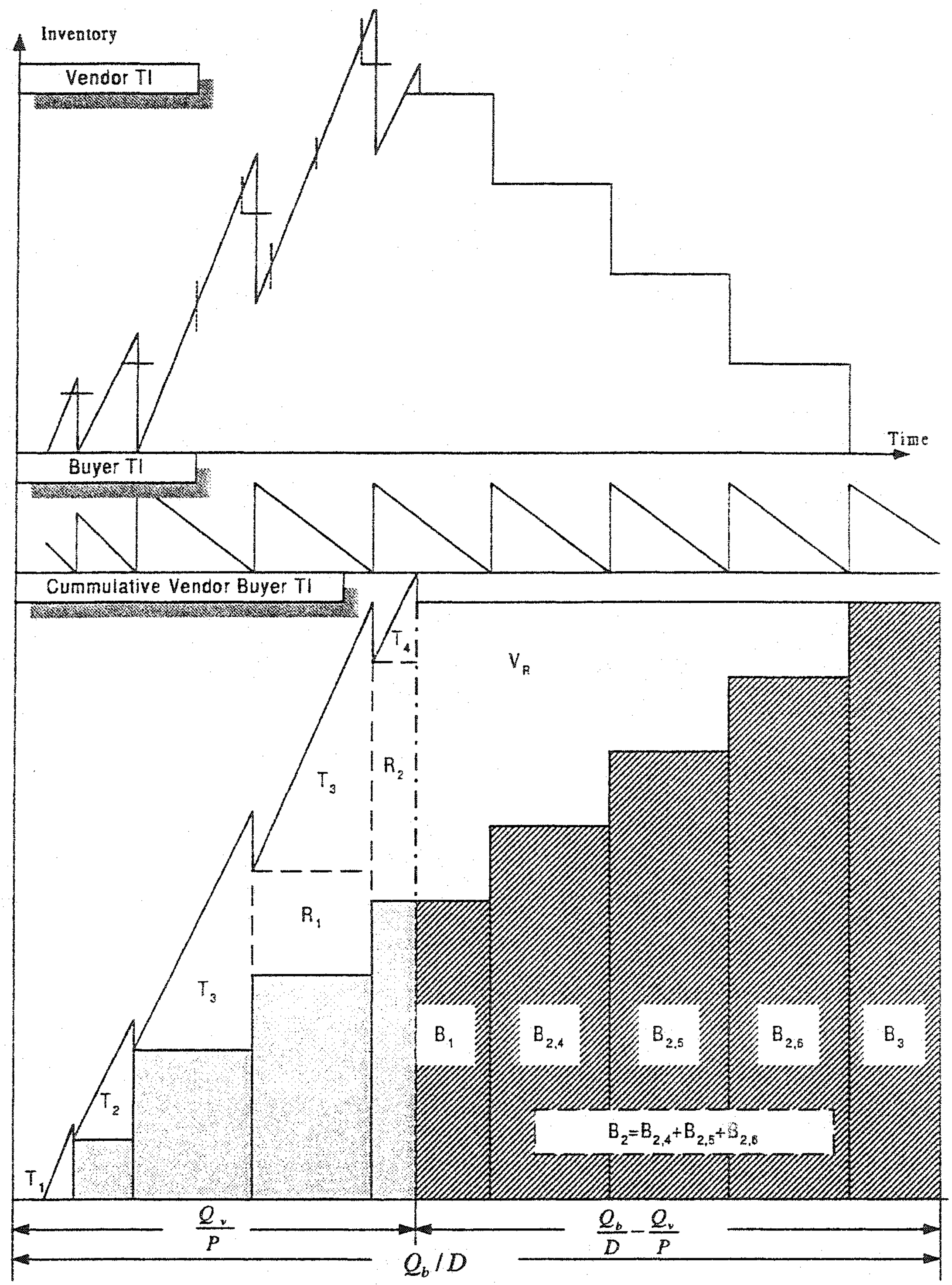

Figure 4.2: The inventory profile of combined vendor buyer system with imperfect production process (when $n = 8$ and $i = 3$) .

We can write the total inventory expression for vendor as;

$$TI_{vendor} = \sum_{l=1}^{4} T_l + \sum_{m=1}^{2} R_m + \left(V_R - \sum_{n=1}^{3} B_n\right) \tag{4.15}$$

The inventory holding cost will be;

$$IHC = \frac{D}{Q_b}[h_v \times TI_{Vendor} + h_b \times TI_{Buyer}] \tag{4.16}$$

When $i = 1$, T_3, R_1 &R_2 becomes 0, and we will have the following, considering Figure (4.3);

$$\begin{aligned} T_1 &= \frac{(q_1)^2}{2P} \\ T_2 &= \frac{(q_2)^2}{2P} \\ T_3 &= \frac{P}{2}[t]^2 \\ V_R &= Q_b\left[\frac{Q_b}{D} - \frac{Q_v}{P}\right] \\ B_1 &= (G_1 + G_2)\left[\frac{G_2}{D} + \frac{G_1}{D} + \frac{q_1}{P} - \frac{Q_v}{P}\right] \end{aligned}$$

if $n - i = 3$ then

$$B_2 = (G_1 + (n-2)G_2)\frac{G_2}{D}$$

if $n - i > 3$ then

$$B_2 = \sum_{j=i+1}^{n-2} (q + jG_2)\frac{G_2}{D}$$

and,

$$B_3 = (G_1 + (n-1)G_2)\left[\frac{G_2}{D} - \frac{q_1}{P}\right]$$

When considering the perfect quality (i.e. when α and θ are zero), equation (4.15) will be reduced to equation (4.4).

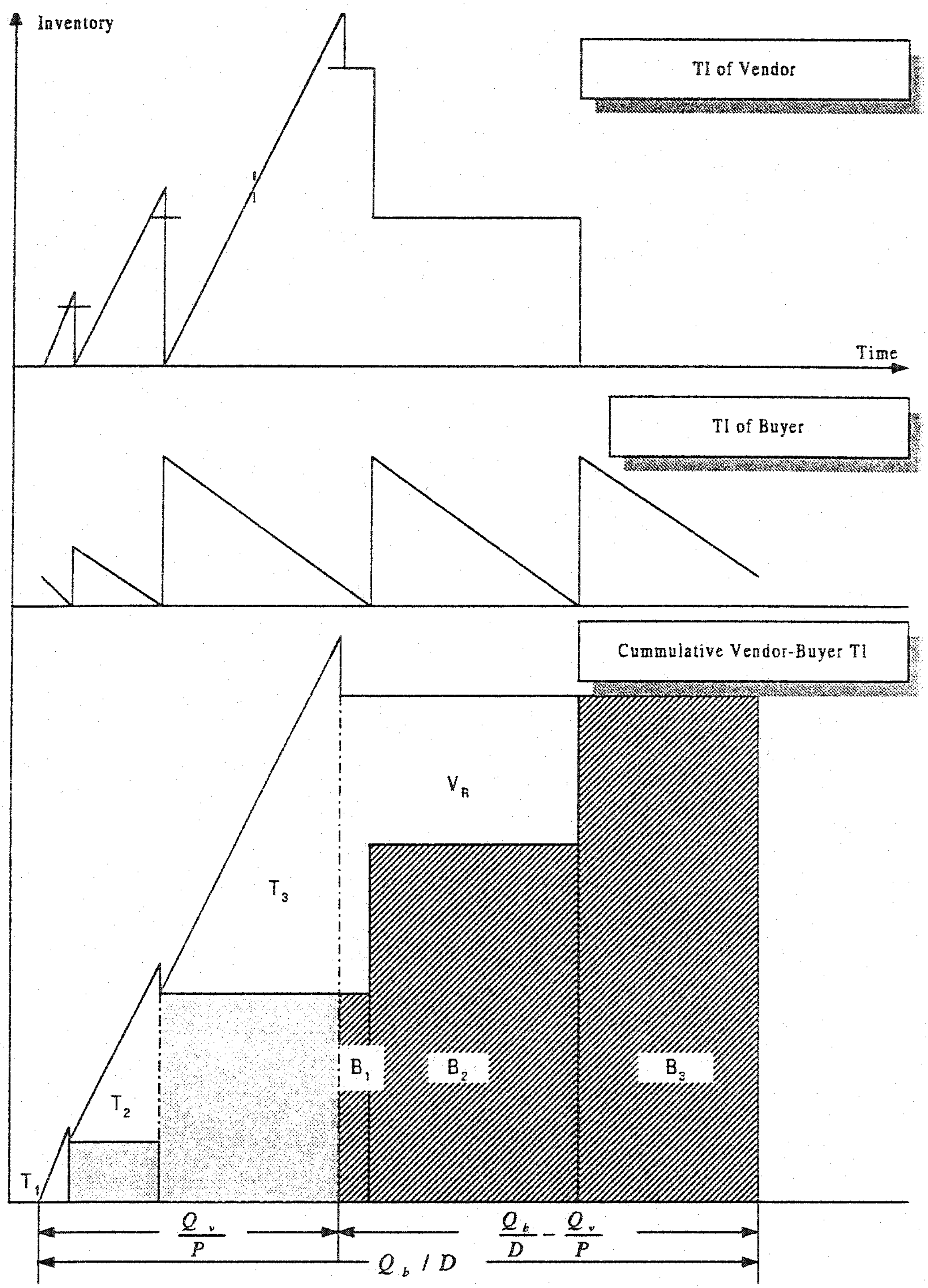

Figure 4.3: The inventory profile of combined vendor buyer system with imperfect production process (when $n = 4$ and $i = 1$).

Expected Restoration Cost

Since restoration is performed after producing each batch in case the process is found in the out of control state, and it is assumed that the restoration cost depends upon the detection delay, then the expected restoration cost after producing the j^{th} batch is given by;

$$E(RC_j) = \int_0^{t_j} \Big(r_0 + r_1\big(t_j - t\big)\Big)\frac{1}{\theta}e^{-t/\theta}dt$$

Consequently the expected restoration cost for the whole system is simply;

$$ERC = \frac{D}{Q_b}\sum_{j=1}^{n}\left[\left(r_0 + r_1\frac{q_j}{P} - r_1\theta\right) + \left(r_1\theta - r_0\right)e^{\frac{-q_j}{P\theta}}\right] \tag{4.17}$$

where, r_0 and r_1 are the restoration cost parameters for the process, and linear relation is assumed between restoration cost and detection delay.

Setup and shipment cost

The production setup cost and the shipment cost incurred by vendor and buyer respectively can be written as:

$$SSC = \frac{(A_v + nA_b)D}{Q_b} \tag{4.18}$$

4.2.2 Total Cost

The expected total cost will be the sum of the quality cost (4.11), the inventory holding cost (4.16), the restoration cost (4.17) and setup and shipment cost (4.18). i.e.

$$ETC = EQC + IHC + SSC + ERC \tag{4.19}$$

Numerical Example

The problem is to find the lot size and the number of shipment that minimizes the total cost. The pattern search technique of Hooke and Jeeves which had been discussed in Section 3.2.2, was used to minimize the cost function. The algorithm was coded and run on a personal computer. Consider the problem given in Goyal [2] with data:

$$\begin{array}{rcl} A_v{=}400 & , & A_b{=}\ 25. \\ h_v{=}4 & , & h_b{=}\ 5(\text{and}\ \ 7). \\ P{=}3200 & , & D = 1000. \end{array}$$

Table 4.1: Problem Parameters

The optimum values of lot size and number of batches calculated in [2] are shown in Table 4.2. Numerical example is presented to illustrate the effect of the developed model and the effects of α and θ are studied and the results are summarized in Table 4.3 and 4.4. Other additional cost parameter are, $s = 60$, $r_0 = 0.03A_v$ and $r_1 = 0.01A_v$.

When $h_b = 5$

n	q	Q	$Cost$
4	52	550	1808

When $h_b = 7$

n	q	Q	$Cost$
5	40	552	1942

Table 4.2: Results for Perfect Production Processes.

The model was run for different values of α, fraction nonconforming when the process is out of control and θ, parameter of the shift distribution to the out of control state. The numerical results for the imperfect joint economic lot size model are summarized in Tables 4.3 and 4.4, which shows the effect of of these parameters on the number of batches, size of each batch, the lot size, the expected quality related cost, restoration cost and total cost. Table 4.3 summarizes the results obtained for the case when $h_b = 5$, whereas Table 4.4 shows the results for $h_b = 7$.

As in [2], it is better to divide the lot into more batches when the holding cost of buyer is high. Compared to the perfect case, smaller batch sizes are produced due to the risk of producing nonconforming items. Total expected costs are higher due to the additional restoration and quality related costs. It can be seen from Figure 4.5 and 4.4 that since restoration is performed after producing the batches so for high rate of nonconforming items, it is better to divide the lot into more batches of smaller size. Hence it can be observed that the number of batches, the batch sizes, the quality and restoration cost and the total expected cost are sensitive to the proportion of nonconforming items produced.

4.2.3 Conclusion

In this section, the effect of deteriorating processes on the joint economic lot size model with batch shipment in developed, where the products are screened out before a shipment is made and the process is restored after producing each batch and if the process is found out in the out of control state. Product and process inspection is assumed to be error free.

For $\alpha = 0.01$

θ	n	q_1	q_2	Q_v	Q_b	$E(QC)$	$E(RC)$	$Cost$
0.075	5	39.71	126.97	547.60	546.43	128.03	39.70	1989.39
0.100	5	40.05	128.07	552.34	551.41	100.66	31.48	1953.01
0.250	4	50.86	162.68	538.89	538.41	53.50	13.79	1875.68
0.500	4	51.45	164.60	545.26	545.01	27.93	7.21	1843.11
0.750	4	51.68	165.36	547.75	547.57	18.91	4.88	1831.66
1.000	4	51.80	165.74	549.03	548.90	14.29	3.69	1825.82

For $\alpha = 0.05$

θ	n	q_1	q_2	Q_v	Q_b	$E(QC)$	$E(RC)$	$Cost$
0.075	9	20.71	66.14	549.82	546.44	370.65	44.29	2366.58
0.100	8	23.37	74.64	545.83	542.97	316.54	33.87	2266.81
0.250	6	32.18	102.86	546.46	544.85	178.00	14.23	2039.93
0.500	5	39.53	126.41	545.17	544.17	109.94	7.28	1939.34
0.750	5	40.23	128.67	554.93	554.23	75.18	4.91	1900.59
1.000	5	40.60	129.87	560.07	559.54	57.14	3.71	1880.73

For $\alpha = 0.10$

θ	n	q_1	q_2	Q_v	Q_b	$E(QC)$	$E(RC)$	$Cost$
0.075	13	14.29	45.57	561.14	556.22	530.72	46.13	2682.81
0.100	11	16.69	53.25	549.15	544.91	466.53	34.97	2534.33
0.250	8	23.73	75.81	554.37	551.90	268.68	14.45	2195.03
0.500	6	32.05	102.46	544.35	542.72	181.06	7.33	2036.78
0.750	6	32.99	105.48	560.36	559.20	124.96	4.93	1973.82
1.000	5	39.47	126.23	544.39	543.39	111.21	3.71	1937.19

For $\alpha = 0.25$

θ	n	q_1	q_2	Q_v	Q_b	$E(QC)$	$E(RC)$	$Cost$
0.075	20	9.08	28.91	558.39	550.40	871.17	47.89	3332.30
0.100	17	10.56	33.64	548.78	541.90	761.09	36.14	3090.86
0.250	12	15.56	49.68	561.99	557.80	450.96	14.70	2529.29
0.500	9	21.04	67.22	558.80	555.98	304.26	7.41	2258.01
0.750	7	26.78	85.56	540.11	537.81	256.34	4.96	2144.13
1.000	7	27.56	88.08	556.06	554.23	198.27	3.73	2078.05

Table 4.3: Effect on different parameters for imperfect Production Processes (having exponential distribution and $h_b = 5$).

For $\alpha = 0.01$

θ	n	q_1	q_2	Q_v	Q_b	$E(QC)$	$E(RC)$	$Cost$
0.075	6	31.90	101.99	541.86	540.90	107.09	41.39	2095.23
0.100	6	32.14	102.79	546.09	545.33	83.55	32.50	2062.19
0.250	6	32.71	104.65	555.96	555.63	36.11	14.18	1995.50
0.500	6	32.97	105.47	560.33	560.16	18.58	7.31	1970.90
0.750	5	38.91	124.48	536.81	536.68	14.54	4.91	1961.59
1.000	5	38.97	124.68	537.68	537.59	10.97	3.70	1956.79

For $\alpha = 0.05$

θ	n	q_1	q_2	Q_v	Q_b	$E(QC)$	$E(RC)$	$Cost$
0.075	10	18.67	59.63	555.31	552.20	337.79	44.79	2427.83
0.100	9	20.74	66.26	550.86	548.26	284.24	34.24	2333.53
0.250	7	27.07	86.54	546.28	544.90	151.67	14.34	2129.12
0.500	6	31.87	101.92	541.45	540.64	89.93	7.32	2044.07
0.750	6	32.30	103.33	548.93	548.37	61.16	4.93	2011.89
1.000	6	32.52	104.04	552.73	552.30	46.34	3.71	1995.46

For $\alpha = 0.10$

θ	n	q_1	q_2	Q_v	Q_b	$E(QC)$	$E(RC)$	$Cost$
0.075	13	14.03	44.77	551.23	546.48	521.84	46.19	2726.80
0.100	12	15.44	49.27	557.39	553.38	433.97	35.15	2584.40
0.250	9	21.04	67.23	558.90	556.67	239.87	14.51	2265.78
0.500	7	27.01	86.34	545.08	543.68	154.03	7.36	2124.84
0.750	7	27.65	88.41	558.11	557.13	105.65	4.94	2070.97
1.000	6	31.83	101.79	540.80	539.98	90.76	3.72	2041.40

For $\alpha = 0.25$

θ	n	q_1	q_2	Q_v	Q_b	$E(QC)$	$E(RC)$	$Cost$
0.075	20	9.01	28.67	553.81	545.95	864.20	47.91	3360.37
0.100	18	10.14	32.31	559.39	552.65	732.21	36.19	3122.85
0.250	12	15.27	48.73	551.29	547.25	442.48	14.71	2577.19
0.500	9	20.45	65.33	543.07	540.40	295.77	7.41	2322.23
0.750	8	23.30	74.45	544.44	542.41	224.36	4.96	2218.07
1.000	8	23.89	76.36	558.38	556.78	172.84	3.73	2160.20

Table 4.4: Effect on different parameters for imperfect Production Processes (having exponential distribution and $h_b = 7$)

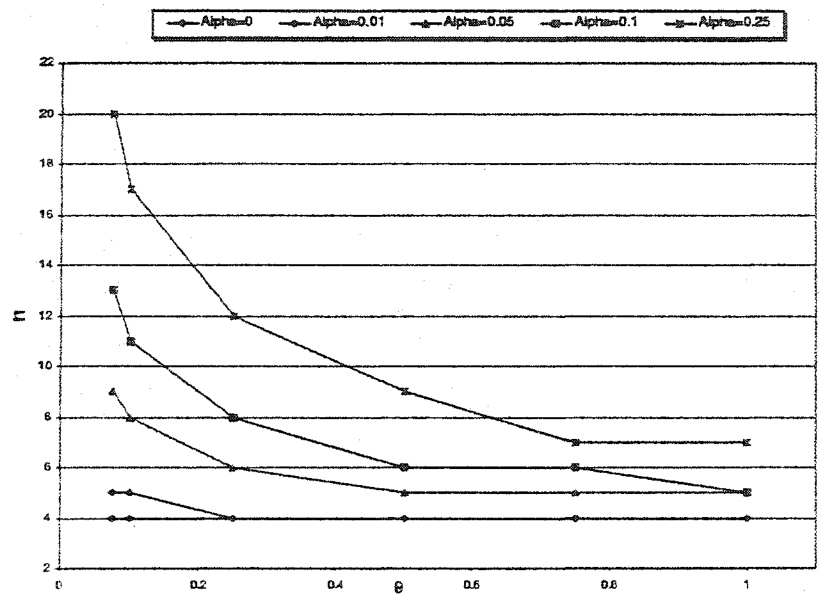

Figure 4.4: Graph between θ and n, with $h_b = 5$

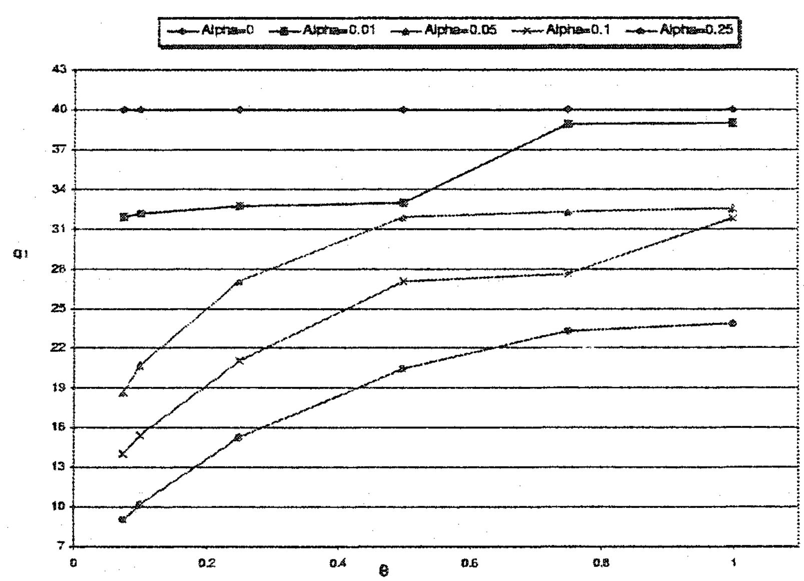

Figure 4.5: Graph between θ and q_1, with $h_b = 7$

4.3 Integrated Imperfect Process Model (without PM)

The purpose of this section is to develop a model that links integrated vendor buyer problem having batch shipment with quality, restoration and maintenance requirements for a process having a general deterioration distribution and where the maintenance level is optimized.

Introduction

The objective of the Preventive maintenance is to prevent, mitigate or detect the onset of failure using diagnostic techniques. In many PM models, the system is assumed to be as good as new after each PM action. However a more realistic situation is one in which the failure pattern of a preventively maintained system changes. One way to model this is to assume that after PM, the failure rate of the system is somewhere between as good as new and as bad as old. This concept is called imperfect maintenance and was introduced by [61] & [62]. It can be assumed that the failure rate of the equipment is decreased after each PM. This amounts to a reduction in the age of the equipment. While developing the model it is assumed that the reduction in the age of the equipment is proportional to the cost of PM. This change in the age of the equipment will affect the time to shift to the out of control state distribution and consequently the amount of nonconforming items. The maintenance level that produces the least total expected cost corresponds to the optimal PM level.

Rosenblat and Lee [56] have found that in classical Economic Production Quantity (EPQ), the production process is subject to a random process deterioration that shifts the system from an in control state to an out of an control state in which it start producing nonconforming items. Lee and Rosenblat [63] also incorporated maintenance by inspection with restoration cost dependent on the detection delay. They assumed that the deterioration of the process is exponentially distributed. Lin et al. [64] extended their work for the case where the deterioration of the process follows a general distribution. Huang and Chiu [65] considered a further extension using a preventive maintenance that either brings the system to the as good as new condition or the system will fail immediately thereafter because of faulty maintenance.

4.3.1 Problem Definition

It is assumed that the production process produces a single item. The production cycle begins with a new system which is assumed to be in an in control state, producing items of acceptable quality. However after a period of time in production, the process may shift to an out of control state. The elapsed time for the process to be in the in control state, before the shift occurs is a random variable assumed to follow a general distribution. After producing each batch, the process is inspected if it is found to be out of control, it is restored. *But this restoration has no effect on the age of the system. It is like a minimal repair.* The assumptions made are same that are discussed in section 4.2.

Notations

The notations used are.

q_1 = Size of the first batch produced by the vendor.

EN_j = Expected number of non conforming items produced in j^{th} interval.

G_1 = The first shipment size received by the buyer, $= q_1 - N_1$.

$q_2, \cdots, q_n$ = Size of the successive batches produced by the vendor (other then the first one). $= G_1 \lambda$.

$G_2, \cdots, G_n$ = Size of the shipment received by the buyer (other then the first one), $q_j - N_j; (j = 2, \cdots, n)$.

t_j = Completion time of production of batch j from the beginning of the cycl

$f(t)$ = Probability density function of the time to shift distribution.

$F(t)$ = cumulative distribution.

$\bar{F}(t)$ = 1-F(t).

Expected number of nonconforming Items (without PM)

Let $E[N_j|t > t_{j-1}]$ represent the nonconforming items produced in the j^{th} interval, provided that the time to shift t is greater then t_{j-1}, i.e.;

$$\begin{aligned} E[N_j|t > t_{j-1}] &= \frac{1}{\bar{F}(t_{j-1})} \int_{t_{j-1}}^{t_j} \alpha(t_j - t) P f(t) dt \\ &= \frac{\alpha P}{\bar{F}(t_{j-1})} \left[t_j \Big(F(t_j) - F(t_{j-1}) \Big) - \int_{t_{j-1}}^{t_j} t f(t) dt \right] \qquad (4.20) \end{aligned}$$

where $F(t_j)$ and $\bar{F}(t_j)$ represent the failure and reliability of the system at interval t_j respectively. For simplicity in notation we will use EN_j instead of $E\left[N_j|t > t_{j-1}\right]$.

Quality Cost

The lot size that the buyer receives Q_b, will be the difference between the lot size produced by the vendor and the nonconforming items that are produced in each batch, i.e.

$$Q_b = Q_v - \sum_{j=1}^{n} EN_j \tag{4.21}$$

Hence the expected quality costs per unit time due to nonconforming items is given by:

$$EQC = \frac{sD}{Q_b} \sum_{j=1}^{n} EN_j \tag{4.22}$$

Quality Control Policy

The nonconforming items are screened out before any shipment is made so in order to determine the vendor inventory area we need to determine the nonconforming items that are produced during consumption of batch j. Let i be the smallest integer such that

$$\frac{Q_v}{P} \leq \sum_{j=1}^{i} \frac{G_j}{D} + \frac{q_1}{P} \tag{4.23}$$

Considering Figure 4.7, let K'_{T_j} represent the number of batches produced during the consumption of batch j which is the ratio between the consumption time of

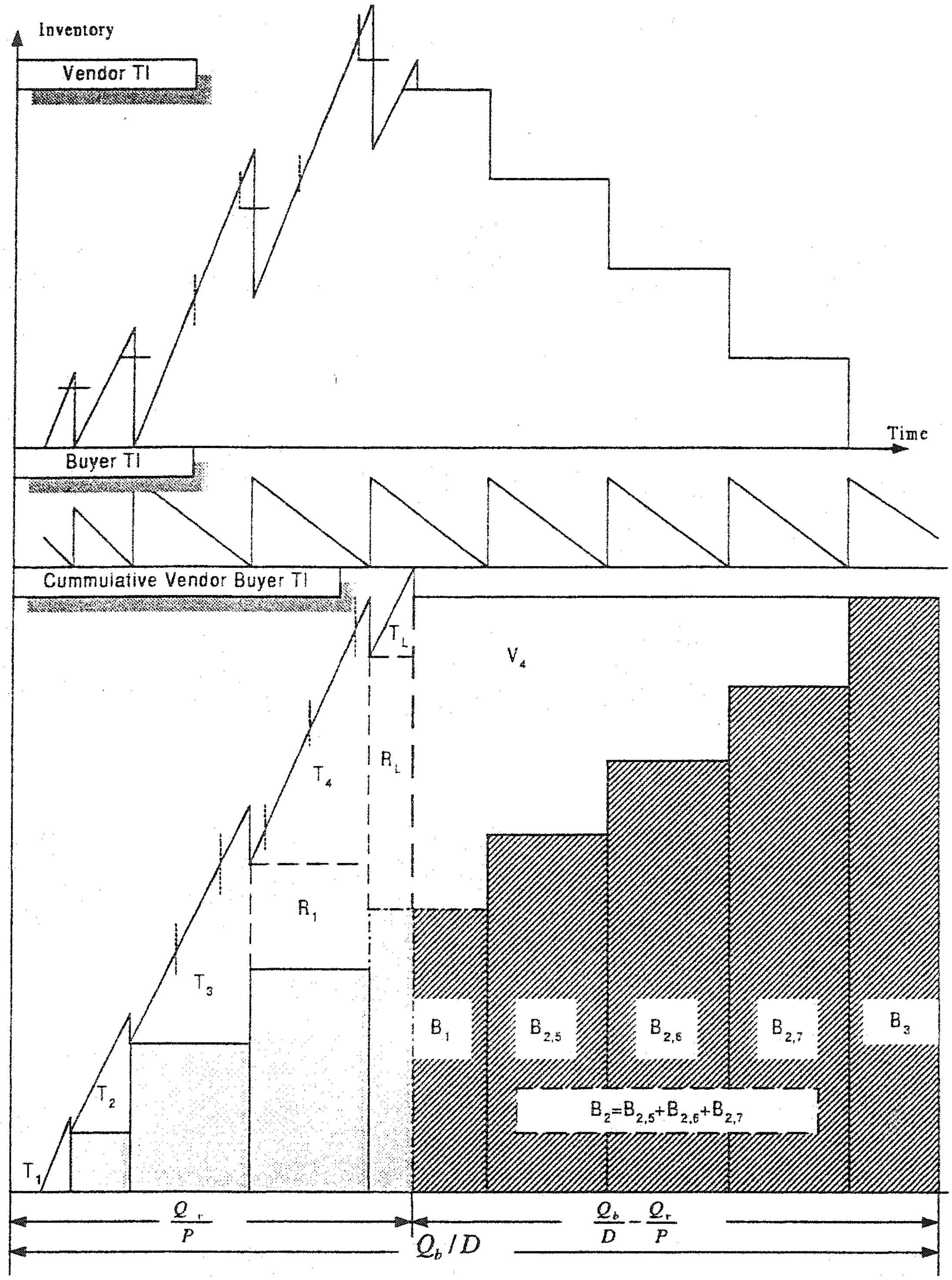

Figure 4.6: The inventory profile of combined vendor buyer system with imperfect production process (when $n = 8$ and $i = 4$).

batch of size G_{j-1} and the production time of batch of size $G_1\lambda$ i.e.

$$\begin{aligned} K'_{T_1} &= K'_{T_2} = 1 \\ K'_{T_j} &= \frac{G_{j-1}/D}{G_1\lambda/P} \quad \text{for } j = 3, \cdots, i \end{aligned} \tag{4.24}$$

Let K_{T_j} represent the number of batches whose production started during the consumption of batch j, also let $I_{K_{T_j}}$ denote the numbers of full batches produced during the consumption of batch j.

$$\begin{aligned} I_{K_{T_j}} &= INT(K_{T_j}) \\ \Delta_{K_{T_j}} &= K_{T_j} - I_{K_{T_j}} \end{aligned}$$

It can be noted from Figure 4.7, that $K_{T_j}+(1-\Delta_{K_{T_{(j-1)}}})$ represent the actual number of batches produced during the interval. Hence, K_{T_j} can be find out recursively as;

$$\begin{aligned} K_{T_3} &= K'_{T_3} \\ K_{T_4} &= K'_{T_4} - (1 - \Delta_{K_{T_3}}) \\ K_{T_5} &= K'_{T_5} - (1 - \Delta_{K_{T_4}}) \\ &\vdots \end{aligned}$$

generally we can write it as for $j = 4, \cdots, i$;

$$K_{T_j} = K'_{T_j} - (1 - \Delta_{K_{T_{j-1}}}) \tag{4.25}$$

Let $\eta_{K_{T_j}}$ represent the number of nonconforming items that are produced during

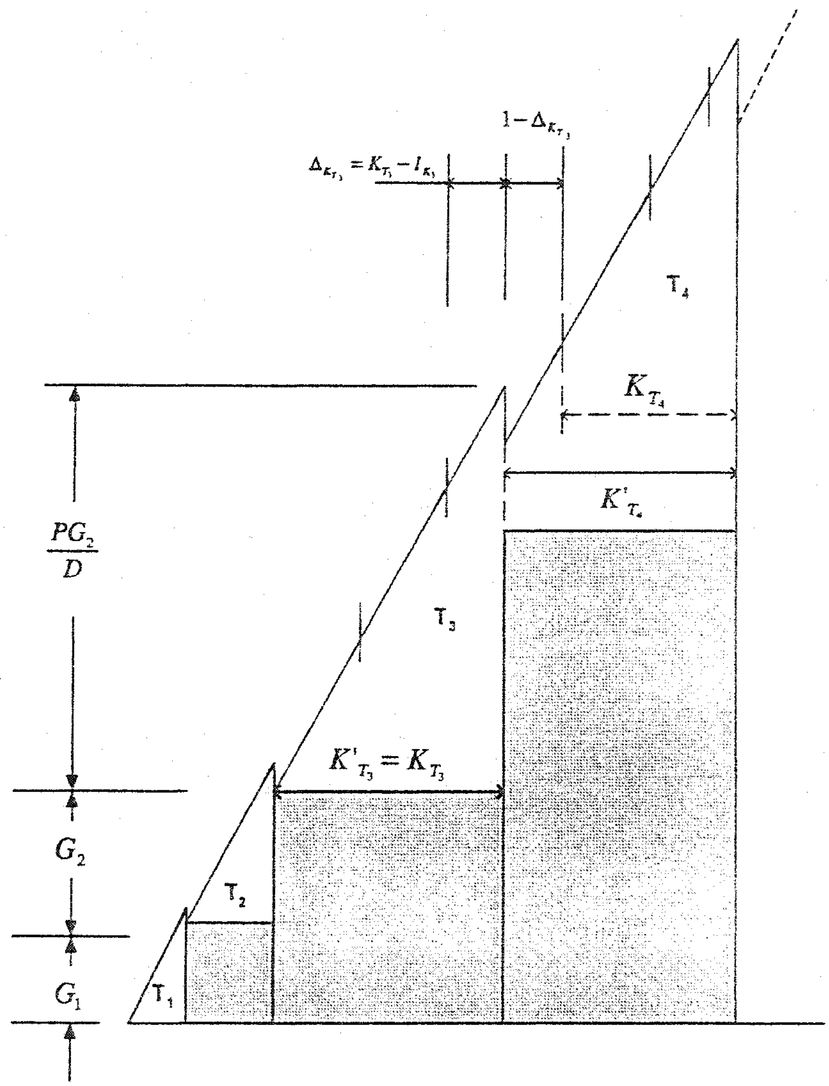

Figure 4.7: Detailed View of Vendor Inventory Profile for general distribution case.

the interval $G_{j-1}/D \ \ (j = 3, \cdots, i)$. i.e.

$$
\begin{aligned}
\eta_{K_{T_3}} &= \sum_{j=3}^{2+I_{K_{T_2}}} EN_j + \Delta_{K_{T_3}} EN_{(3+I_{K_{T_3}})} \\
\eta_{K_{T_4}} &= (1-\Delta_{K_{T_3}}) EN_{(3+I_{K_{T_3}})} + \sum_{j=4+I_{K_{T_3}}}^{3+I_{K_{T_3}}+I_{K_{T_4}}} EN_j \\
&+ \Delta_{K_{T_4}} EN_{(4+I_{K_{T_3}}+I_{K_{T_4}})} \\
\eta_{K_{T_5}} &= (1-\Delta_{K_{T_4}}) EN_{(4+I_{K_{T_3}}+I_{K_{T_4}})} + \sum_{j=5+I_{K_{T_3}}+I_{K_{T_4}}}^{4+I_{K_{T_3}}+I_{K_{T_4}}+I_{K_{T_5}}} EN_j \\
&+ \Delta_{K_{T_5}} EN_{(5+I_{K_{T_3}}+I_{K_{T_4}}+I_{K_{T_5}})} \\
&\vdots
\end{aligned}
$$

In general we can write it as; for $j = 4, \cdots, i$

$$
\begin{aligned}
\eta_{K_{T_j}} &= (1-\Delta_{K_{T_{j-1}}}) EN_{(j-1+S_{j-1})} + \sum_{l=j+S_{j-1}}^{j-1+S_j} EN_l \\
&+ \Delta_{K_{T_j}} EN_{(j+S_j)}
\end{aligned} \tag{4.26}
$$

where, $S_l = \sum_{j=3}^{l} I_{K_{T_j}}$.

Inventory Holding Cost of the system

Considering Figure 4.6, the *total inventory area of vendor* is found out by adding the areas of different rectangles and triangles, i.e.

$$T_1 = \frac{q_1^2}{2P} \tag{4.27}$$

$$T_2 = \frac{q_2^2}{2P} \tag{4.28}$$

$$T_j = \frac{P}{2}\left(\frac{G_{(j-1)}}{D}\right)^2 \quad \text{for } j = 3, \cdots, i \tag{4.29}$$

$$T_L = \frac{P}{2}\left[\frac{Q_v}{P} - \left(\sum_{j=1}^{i-1}\frac{G_j}{D} + \frac{q_1}{P}\right)\right]^2 \tag{4.30}$$

$$R_L = \left[\frac{Q_v}{P} - \left(\sum_{j=1}^{i-1}\frac{G_j}{D} + \frac{q_1}{P}\right)\right]\left[P\sum_{j=1}^{i-2}\frac{G_{j+1}}{D} - \sum_{j=1}^{i-2}\left(\eta_{K_{T_{(j+2)}}} + G_{(j+2)}\right)\right] \tag{4.31}$$

$$V_4 = \sum_{m=1}^{i-3} R_m$$

where,

$$R_m = \left[P\sum_{j=1}^{m}\frac{G_{j+1}}{D} - \sum_{j=1}^{m}\left(\eta_{K_{T_{(j+2)}}} + G_{(j+2)}\right)\right]\frac{G_{m+2}}{D} \quad \text{for } m = 1, .., i-3 \tag{4.32}$$

$$V_5 = V_R = Q_b\left[\frac{Q_b}{D} - \frac{Q_v}{P}\right]$$

$$B_1 = \sum_{j=1}^{i} G_j\left[\sum_{j=1}^{i}\frac{G_j}{D} + \frac{q_1}{P} - \frac{Q_v}{P}\right]$$

where i is defined by equation (4.23).

if $n - i = 2$,

$$B_2 = \frac{G_{n-1}}{D}\sum_{j=1}^{n-1} G_j$$

if $n - i > 2$,

$$B_2 = \sum_{m=i+1}^{n-1} B_{2,m}$$

where,

$$B_{2,m} = \frac{G_m}{D}\sum_{j=1}^{m} G_j \quad \text{for } m = i+1, \cdots, n-1$$

$$B_3 = Q_b\left[\frac{G_n}{D} - \frac{q_1}{P}\right]$$

Representing equation (4.27) and (4.28) as,

$$V_1 = T_1 + T_2 \tag{4.33}$$

equation (4.29) as,

$$V_2 = \sum_{j=3}^{i} \frac{P}{2} \left(\frac{G_{(j-1)}}{D} \right)^2$$

and equation (4.30) and (4.31) by;

$$V_3 = T_L + R_L$$

We can now determine the Total Inventory of the vendor as; (the unshaded area in Figure 4.6).

$$TI_{Vendor} = \sum_{s=1}^{5} V_s - \sum_{t=1}^{3} B_t \tag{4.34}$$

Total Inventory of Buyer

The Total Inventory of buyer will simply be the sum of the area of all the shipments triangle, i.e.

$$\begin{aligned} TI_{Buyer} &= \frac{1}{2D} \sum_{j=1}^{n} \Big(q_j - E(N_j) \Big)^2 \\ &= \frac{1}{2D} \sum_{j=1}^{n} (G_j)^2 \end{aligned} \tag{4.35}$$

The inventory holding cost per unit time will be;

$$IHC = \frac{D}{Q_b} [h_v \times TI_{Vendor} + h_b \times TI_{Buyer}] \tag{4.36}$$

Expected Restoration Cost (without PM)

It is assumed that the restoration is performed after producing batches and it doesn't have any effect on the age of the system. The restoration cost depends upon the detection delay, so the expected restoration cost during the j^{th} interval is given by;

$$\begin{aligned} E\big[RC_j|t > t_{j-1}\big] &= \frac{1}{\bar{F}(t_{j-1})}\int_{t_{j-1}}^{t_j} R(t_j - t)f(t)dt \\ &= \frac{1}{\bar{F}(t_{j-1})}\left[\begin{array}{c} \Big(r_0 + r_1 t_j\Big)\Big\{F(t_j) - F(t_{j-1})\Big\} \\ -r_1 \int_{t_{j-1}}^{t_j} tf(t)dt \end{array}\right] \end{aligned} \quad (4.37)$$

The second equality is obtained by assuming that the restoration cost change linearly with the detection delay. i.e.

$$R(t_j - t) = r_0 + r_1(t_j - t)$$

where, r_0 and r_1 are constants. The expected restoration cost per unit time is given by;

$$ERC = \frac{D}{Q_b}\sum_{j=1}^{n} E\big[RC_j|t > t_{j-1}\big] \quad (4.38)$$

Setup and shipment cost

The production setup cost and the shipment cost incurred by vendor and buyer respectively can be written as:

$$SSC = \frac{(A_v + nA_b)D}{Q_b} \quad (4.39)$$

4.3.2 Total Cost (without PM)

The expected total cost will be the sum of the quality cost (4.22), the inventory holding cost (4.36), the restoration cost (4.38) and setup and shipment cost (4.39). i.e.

$$ETC = EQC + IHC + SSC + ERC \tag{4.40}$$

4.3.3 Integrated Imperfect Process Model (with PM)

It is now assumed that PM (Preventive Maintenance) is carried out after producing the batches along with restoration and the effect of PM activities on the deterioration pattern of the process is modeled using the imperfect maintenance concept. The objective of PM activities is to prevent or delay the onset of failure or shift in the process. It is considered that, after each PM the failure rate of the equipment is decreased this corresponds to the reduction in age of the system which is reduced proportional to the PM level. The amount of PM efforts which results in the least expected total cost corresponds to the optimal PM level. The total expected cost will now consists of the setup cost, inventory holding cost, quality cost due to the production of nonconforming items, PM cost, and restoration cost. Following additional assumptions are made;

1. PM is done after producing each batch.

2. Each PM reduces the age of the equipment by a factor which is proportional to the level of PM done.

3. The level of PM is kept constant throughout the time horizon under consideration.

And the following additional notations are used;

h_j = Length of the j^{th} inspection interval or production time of batch j.

y_j = Actual age of the system right before the j^{th} PM.

w_j = Actual age of the system right after the j^{th} PM.

C_{pm} = Cost of actual PM activities.

C^0_{pm} = Cost of maximum PM level.

Determining the age of the system with PM

As mentioned earlier the concept of imperfect maintenance is used, i.e. after each PM, the age of the system is somewhere between as good as new and as bad as old depending on the level of PM activities. The reduction in the age of the equipment is a function of the cost of preventive maintenance. Let y_k (w_k) represent the age of the system just before (after) the k^{th} PM. Let

$$\gamma_k = \xi^{k-1}\frac{C_{pm}}{C^0_{pm}} \tag{4.41}$$

where, $0 < \ \xi\ > 1$, is an imperfectness factor which implies that there is a degradation in the effect of PM on the age of the system. A full PM brings the system farther from the as good as new condition as more PMs are performed. Linear and nonlinear problems between age reduction and PM cost may be considered [66].

Here we assume that this relationship is linear and given by;

$$w_k = (1 - \gamma_k) y_k \tag{4.42}$$

The effective age of the equipment at time t_j is given by;

$$\begin{aligned} y_1 &= h_v \\ y_2 &= w_1 + h_b \\ &\vdots \\ y_j &= w_{j-1} + h_j \end{aligned} \tag{4.43}$$

This change in the age of the equipment due to PM will affect the number of nonconforming items, restoration costs and on number of batches and consequently allows the joint optimization of production quantity, quality control costs, and the cost of preventive maintenance. Thus production, quality and maintenance aspects are integrated in one model. Because of this change in age of the system the expression for determining the expected number of nonconforming items, and expected restoration cost will also change and they will now calculated by using the following expressions;

Expected Number of nonconforming items (with PM)

Let $E[N_j|t > w_{j-1}]$ represent the nonconforming items produced in the j^{th} interval, provided that the time to shift t is greater then w_{j-1};

$$\begin{aligned} E\big[N_j|t > w_{j-1}\big] &= \frac{1}{\bar{F}(w_{j-1})}\int_{w_{j-1}}^{y_j} \alpha(y_j - t)Pf(t)dt \\ &= \frac{\alpha P}{\bar{F}(w_{j-1})}\left[y_j\Big(F(y_j) - F(w_{j-1})\Big) - \int_{w_{j-1}}^{y_j} tf(t)dt\right] \end{aligned} \tag{4.44}$$

For simplicity it is assumed that $EN_j = E\big[N_j|t > w_{j-1}\big]$

Expected Quality Cost (with PM)

The lot size that the buyer receives Q_b, will be the difference between the lot size produced by the vendor and the nonconforming items that are produced in each batch, i.e.

$$Q_b = Q_v - \sum_{j=1}^{n} EN_j \tag{4.45}$$

The expected quality cost per unit time is:

$$EQC = \frac{D}{Q_b} s \sum_{j=1}^{n} EN_j \tag{4.46}$$

Expected Restoration Cost (after PM)

The expected cost of restoration done at the end of j^{th} inspection interval is given by;

$$\begin{aligned} E\big[RC_j|t > w_{j-1}\big] &= \frac{1}{\bar{F}(w_{j-1})}\int_{w_{j-1}}^{y_j} R(y_j - t)f(t)dt \\ &= \frac{1}{\bar{F}(w_{j-1})}\left[\begin{array}{c} \Big(r_0 + r_1 y_j\Big)\Big\{F(y_j) - F(w_{j-1})\Big\} \\ -r_1 \int_{w_{j-1}}^{y_j} tf(t)dt \end{array}\right] \end{aligned} \tag{4.47}$$

Hence the expected Restoration cost of the whole system will be

$$ERC = \frac{D}{Q_b} \sum_{j=1}^{n} E\left[RC_j | t > w_{j-1}\right] \tag{4.48}$$

PM Costs

Since it is assumed that PM activities are carried out after producing each batch, so the total PM cost per unit time is given by;

$$PMC = \frac{D}{Q_b}\, n\, C_{pm} \tag{4.49}$$

4.3.4 Total Cost (with PM)

The total cost will now be the sum of equations (4.46), (4.36), (4.39), (4.48) and (4.49).

Numerical Example

The problem is to determine the value of the decision variables; n, q_1, that minimizes the expected total cost ETC. Consider the problem given in [2], the data is shown in Table 4.1. The optimum values of lot size and number of batches calculated in [2] are shown in Table 4.2. It is assumed that the process shift mechanism is assumed to follow weibull distribution with scale parameter θ and shape parameter β, so the probability density function is given by;

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}} \tag{4.50}$$

Hence the failure and the reliability of the system at interval t_j are determined as;

$$\begin{aligned} F(t_j) &= 1 - exp\left[-\left(\frac{t_j}{\theta}\right)^{\beta}\right] \\ \bar{F}(t_j) &= 1 - F(t_j) = exp\left[-\left(\frac{t_j}{\theta}\right)^{\beta}\right] \end{aligned}$$

The size of the shipments, the number of shipment and cost parameters at different PM levels corresponding to $C^0_{pm} = 10$ are calculated. The cost function is minimized using the pattern search technique of Hooke and Jeeves [60]. The results are summarized in Tables 4.5, 4.6, 4.7, 4.8 and in Fig. 4.8, 4.9, 4.10, 4.11 and 4.12 for $h_b = 5$, and for $h_b = 7$ in Tables 4.9, 4.10, 4.11, 4.12 and in Figures 4.13, 4.14, 4.15, 4.16 and 4.17 for different Weibull shape parameter i.e $\beta = 1.25, 2.25$ and 3.25, respectively and for a range of μ which is the mean time to shift. Same quality parameters are used, i.e.; $s = 60$, $\tau_0 = 0.03A_v$ and $\tau_1 = 0.01A_v$.

On comparing these tables and figures which illustrate clearly the trade-offs between PM levels and quality related costs, the increase in PM level yields reduction in quality control costs, as visible in Figure 4.18. Also the PM activities effect the economic production quantity (EPQ). Therefore, it can be concluded that if the cost of performing PM is high to the point where it is not compensated for by the reductions in the quality related costs then performing PM is not justifiable, as it can be seen in Figure 4.12 and 4.17. Beside this we can also notice from Figure 4.19, that PM does affect the production cycle and for higher PM levels we have longer production cycles. This is because when PMs are performed during the production cycle quality related costs reduced, which result in longer production cycle time.

4.3.5 Conclusion

In this section we developed the generalized cost models for integrated vendor buyer system by incorporating quality and maintenance functions. We consider two cases, in the first case it is assumed that only restoration is done after producing each batch but it doesn't have any effect on the age of the system, then in the second case we assumed that PM is done along with restoration, which have its effect on the age of the system.

When No Age Reduction

For $\alpha = 0.01$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	$TotalCost$
0.10	5	39.42	126.10	543.82	542.89	102.41	32.26	1957.09
0.30	4	50.95	163.02	540.00	539.68	36.06	9.34	1853.72
0.60	4	51.62	165.18	547.16	547.02	15.82	4.09	1827.18
0.90	4	51.85	165.93	549.63	549.55	9.66	2.50	1819.99

For $\alpha = 0.05$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	$TotalCost$
0.10	8	22.45	71.780	524.91	522.18	314.05	34.83	2277.75
0.30	5	38.20	122.21	527.04	525.82	138.37	9.44	1974.70
0.60	5	40.12	128.35	553.53	552.95	62.96	4.13	1887.80
0.90	4	50.58	161.82	536.04	535.62	46.84	2.48	1858.02

For $\alpha = 0.10$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	$TotalCost$
0.10	10	16.83	53.80	501.05	497.08	479.20	35.53	2551.47
0.30	7	26.83	85.80	541.65	539.84	201.12	9.63	2092.43
0.60	5	38.62	123.54	532.79	531.73	120.25	4.09	1949.47
0.90	5	39.77	127.24	548.72	548.03	75.52	2.50	1899.50

For $\alpha = 0.25$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	$TotalCost$
0.10	15	10.49	33.53	479.90	473.86	765.10	36.46	3115.49
0.30	9	19.43	62.13	516.51	513.37	366.38	9.63	2352.62
0.60	7	26.61	85.11	537.24	535.35	212.29	4.13	2100.21
0.90	6	31.82	101.77	540.68	539.31	152.43	2.50	2004.40

Table 4.5: Results when $\beta = 1.25$, $h_b = 5$ and *No Age Reduction*

<u>*When* $C_{pm}/C^0_{pm} = 0.1$</u>

For $\alpha = 0.01$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	5	39.65	126.82	564.94	546.03	100.14	31.52	9.16	1962.62
0.30	4	51.18	163.74	542.40	542.08	35.34	9.15	7.38	1860.01
0.60	4	51.84	165.86	549.42	549.27	15.49	4.01	7.28	1834.61
0.90	4	52.05	166.53	551.64	551.55	9.45	2.44	7.25	1826.94

For $\alpha = 0.05$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	8	22.75	72.75	532.01	529.33	303.93	33.42	15.11	2276.49
0.30	5	38.48	123.08	530.78	529.58	135.13	9.19	9.44	1979.49
0.60	5	40.36	129.11	556.79	556.22	61.37	4.01	8.99	1894.65
0.90	4	50.81	162.57	538.52	538.10	45.88	2.43	7.43	1864.19

For $\alpha = 0.10$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	10	17.16	54.86	510.88	506.98	461.38	33.70	19.72	2540.56
0.30	6	30.73	98.26	522.05	520.16	217.42	9.18	11.53	2095.58
0.60	5	38.90	124.42	536.56	535.51	117.38	3.98	9.34	1954.80
0.90	5	40.02	128.04	552.19	551.51	73.63	2.43	9.07	1906.05

For $\alpha = 0.25$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	16	10.49	33.52	513.26	507.28	707.84	33.87	31.54	3074.57
0.30	9	19.78	63.24	525.71	522.63	353.42	9.16	17.22	2348.55
0.60	7	26.90	86.02	543.01	541.16	205.47	3.97	12.94	2103.43
0.90	6	32.10	102.66	545.41	544.07	148.10	2.42	11.03	2009.42

Table 4.6: Results when $\beta = 1.25$, $h_b = 5$ and *PM ratio is* 0.1

$\underline{When\ C_{pm}/C^0_{pm} = 0.5}$

For $\alpha = 0.01$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	5	40.62	129.93	560.34	559.50	90.10	28.42	44.68	1983.09
0.30	4	52.11	166.71	552.25	551.95	32.04	8.32	36.24	1884.37
0.60	4	52.71	168.65	558.65	558.52	13.98	3.63	35.81	1861.30
0.90	4	52.92	169.33	560.91	560.83	8.52	2.21	35.66	1854.36

For $\alpha = 0.05$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	7	26.43	84.49	533.36	530.78	292.27	28.43	65.94	2272.53
0.25	5	39.61	126.70	546.43	545.33	120.93	8.16	45.84	1996.85
0.50	4	50.94	162.96	539.83	539.23	67.09	3.61	37.09	1916.11
0.75	4	51.80	165.72	548.95	548.57	41.52	2.20	36.46	1888.00

For $\alpha = 0.10$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	10	18.43	58.89	548.41	544.79	398.90	27.82	91.78	2510.83
0.25	6	31.98	102.26	543.28	541.54	192.65	7.99	55.40	2104.64
0.50	5	40.01	127.97	551.89	550.93	104.71	3.53	45.38	1974.57
0.75	4	50.49	161.52	535.03	534.32	80.49	2.19	37.43	1928.92

For $\alpha = 0.25$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	15	12.06	38.52	551.35	545.69	622.13	26.53	137.44	2987.87
0.25	8	22.81	72.92	533.26	530.33	332.07	7.66	75.42	2338.42
0.50	6	31.71	101.41	538.77	536.95	203.18	3.42	55.87	2112.67
0.75	5	38.76	123.98	534.68	533.32	152.50	2.13	46.88	2026.14

Table 4.7: Results when $\beta = 1.25$, $h_b = 5$ and *PM ratio is* 0.5

When $C_{pm}/C^0_{pm} = 1.0$

For $\alpha = 0.01$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	4	51.57	164.93	546.36	545.55	88.77	24.03	73.32	1993.98
0.30	4	53.34	170.67	565.36	565.12	25.65	7.01	70.78	1911.62
0.60	4	53.83	172.25	570.57	570.47	11.12	3.04	70.12	1892.92
0.90	3	72.35	231.50	535.35	535.27	9.32	1.95	56.05	1886.10

For $\alpha = 0.05$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	6	31.68	101.26	538.00	535.62	266.09	23.41	112.02	2252.05
0.25	5	41.21	131.82	568.49	567.58	96.09	6.72	88.09	2010.13
0.50	4	52.38	167.56	555.07	554.57	53.76	3.02	72.13	1936.65
0.75	4	53.08	169.83	562.55	562.24	33.10	1.84	71.14	1914.08

For $\alpha = 0.10$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	8	23.19	74.09	541.82	538.43	377.58	22.63	148.58	2466.95
0.25	5	39.10	125.02	539.16	537.54	180.40	6.65	93.02	2103.56
0.50	4	50.75	162.33	537.75	536.83	103.48	3.00	74.51	1989.48
0.75	4	52.03	166.43	551.30	550.71	64.59	1.83	72.63	1946.79

For $\alpha = 0.25$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	12	15.15	48.36	547.09	541.81	584.75	21.35	221.48	2904.61
0.25	7	27.02	86.34	545.05	542.38	295.63	6.23	129.06	2311.12
0.50	5	38.79	124.03	534.92	533.23	190.31	2.85	93.77	2111.51
0.75	5	40.53	129.61	558.98	557.85	121.41	1.74	89.63	2032.78

Table 4.8: Results when $\beta = 1.25$, $h_b = 5$ and *PM ratio is* 1.0

Beta=2.25 $\underline{h_2=5}$

No Age Reduction

α=0.01

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	TC
0.1	4	45.97	147.09	487.25	486.25	123.33	794.58	1028.3	33.53	1979.7
0.3	4	51.03	163.29	540.89	540.74	16.37	883.62	924.66	4.48	1829.1
0.6	4	51.92	166.13	550.31	550.28	3.66	899.2	908.63	1	1812.5
0.9	4	52.07	166.62	551.92	551.91	1.48	901.86	905.95	0.4	1809.7

α=0.05

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	TC
0.1	6	24.54	78.53	417.19	414.95	324.47	638.33	1325.5	33.61	2321.9
0.3	4	47.79	152.93	506.57	505.97	70.96	826.8	988.21	4.15	1890.1
0.6	4	51.05	163.35	541.1	540.95	17.62	883.95	924.31	0.98	1826.9
0.9	4	51.71	165.46	548.09	548.02	7.31	895.51	912.37	0.4	1815.6

α=0.10

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	TC
0.1	7	18.52	59.26	374.06	371.27	450.24	561.53	1548.7	31.12	2591.6
0.3	5	36.49	116.75	503.49	502.56	111.33	791.93	1044.7	4.15	1952.1
0.6	4	50.04	160.13	530.42	530.12	33.71	866.27	943.18	0.95	1844.1
0.9	4	51.25	164.01	543.29	543.17	14.33	887.58	920.53	0.4	1822.8

α=0.25

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	TC
0.1	9	12.17	38.93	323.63	320.15	650.7	473.48	1952.2	27.43	3103.8
0.3	6	27.69	88.59	470.66	469.1	198.94	721.74	1172.5	3.85	2097
0.6	4	47.58	152.25	504.34	503.7	75.33	823.1	992.65	0.89	1892
0.9	4	50.04	160.13	530.42	530.12	33.96	866.26	943.18	0.38	1843.8

θ=0.1129, 0.3387, 0.6774, 1.0161

Figure 4.8: Results when $\beta = 2.25$,$h_b = 5$, and *No Age Reduction*

Beta=2.25 h_2=5

PM Ratio=0.1

α=0.01

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	5	37.59	120.28	518.71	517.86	98.47	816.04	1013.8	33.32	9.66	1971.3
0.3	4	51.36	164.35	544.42	544.29	14.31	889.42	918.62	3.96	7.35	1833.7
0.6	4	52.16	166.9	552.85	552.82	3.17	903.36	904.45	0.87	7.24	1819.1
0.9	4	52.3	167.36	554.37	554.36	1.29	905.87	901.94	0.35	7.22	1816.7

α=0.05

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	7	22.7	72.64	458.53	456.5	266.46	690.58	1259.6	30.34	15.33	2262.3
0.3	4	48.47	155.1	513.77	513.24	63.02	838.67	974.21	3.7	7.79	1887.4
0.6	4	51.38	164.39	544.55	544.42	15.34	889.62	918.42	0.86	7.35	1831.6
0.9	4	51.98	166.34	550.99	550.93	6.34	900.27	907.55	0.35	7.26	1821.8

α=0.10

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	9	16.53	52.89	439.67	437.05	358.6	646.62	1430	28.06	20.59	2483.9
0.3	5	37.34	119.49	515.3	514.47	96.7	810.7	1020.5	3.59	9.72	1941.2
0.6	4	50.51	161.61	535.34	535.07	29.54	874.36	934.45	0.84	7.48	1846.7
0.9	4	51.6	165.12	546.95	546.84	12.48	893.57	914.35	0.35	7.31	1828.1

α=0.25

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	12	11.22	35.89	406.04	402.57	516.83	584.08	1738.8	23.84	29.81	2893.4
0.3	6	28.78	92.07	489.15	487.75	171.59	750.44	1127.6	3.25	12.3	2065.2
0.6	4	48.26	154.43	511.55	510.99	66.74	835	978.5	0.79	7.83	1888.9
0.9	4	50.51	161.61	535.34	535.07	29.74	874.35	934.45	0.34	7.48	1846.4

θ=0.1129, 0.3387, 0.6774, 1.0161

Figure 4.9: Results when $\beta = 2.25$,$h_b = 5$, and *PM Ratio=0.1*

Beta=2.25 h_2=5

PM Ratio=0.5

α=0.01

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	5	39.96	127.84	551.33	550.84	53.77	867.97	953.09	19.73	45.39	1939.9
0.3	4	52.65	168.48	558.08	558.01	7.71	911.82	896.05	2.3	35.84	1853.7
0.6	4	53.1	169.91	562.83	562.81	1.67	919.68	888.39	0.5	35.54	1845.8
0.9	4	53.19	170.18	563.73	563.72	0.67	921.17	886.96	0.2	35.48	1844.5

α=0.05

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	7	26.75	85.59	540.28	539.05	137.54	815.54	1066.7	15.07	64.93	2099.8
0.3	4	50.93	162.95	539.77	539.45	35.81	881.49	926.88	2.21	37.08	1883.5
0.6	4	52.69	168.6	558.48	558.4	8.2	912.47	895.42	0.49	35.82	1852.4
0.9	4	53	169.6	561.79	561.76	3.34	917.96	890.06	0.2	35.6	1847.2

α=0.10

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	8	22.34	71.46	522.54	520.83	196.86	778.16	1152	12.91	76.8	2216.7
0.3	5	40.3	128.96	556.13	555.67	50.04	875.58	944.81	1.9	44.99	1917.3
0.6	4	52.18	166.96	553.07	552.92	16.04	903.51	904.29	0.49	36.17	1860.5
0.9	4	52.8	168.96	559.66	559.6	6.62	914.43	893.49	0.2	35.74	1850.5

α=0.25

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	11	16.07	51.42	530.28	527.93	267.27	770.23	1278.6	9.68	104.18	2429.9
0.3	5	37.64	120.42	519.33	518.4	107.46	816.85	1012.7	1.75	48.23	1987
0.6	4	50.84	162.66	538.81	538.47	37.83	879.9	928.55	0.47	37.14	1883.9
0.9	4	52.19	167.01	553.22	553.07	16.14	903.76	904.04	0.2	36.16	1860.3

θ=0.1129, 0.3387, 0.6774, 1.0161

Figure 4.10: Results when $\beta = 2.25, h_b = 5$, and *PM Ratio=0.5*

Beta=2.25 h_2=5

PM Ratio=1.0

α=0.01

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	4	52.16	166.91	552.88	552.61	29.08	902.98	904.79	11.07	72.38	1920.3
0.3	3	72.31	231.39	535.08	535.04	5.04	930.99	887.78	1.44	56.07	1881.3
0.6	3	72.71	232.67	538.06	538.05	1.08	936.23	882.82	0.31	55.76	1876.2
0.9	3	72.75	232.79	538.33	538.33	0.43	936.72	882.36	0.12	55.73	1875.4

α=0.05

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	5	39.62	126.77	546.72	545.96	83.35	860.2	961.61	8.4	91.58	2005.1
0.3	4	53.35	170.72	565.51	565.38	13.54	923.86	884.36	1.04	70.75	1893.5
0.6	3	72.31	231.37	535.04	534.99	5.33	930.9	887.87	0.31	56.08	1880.5
0.9	3	72.6	232.32	537.25	537.23	2.16	934.81	884.16	0.12	55.84	1877.1

α=0.10

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	6	32.54	104.11	553.07	552.05	111.38	849.28	996.29	6.82	108.69	2072.5
0.3	4	52.55	168.14	556.98	556.74	26.17	909.72	898.09	1.02	71.85	1906.8
0.6	4	53.84	172.29	570.71	570.66	5.83	932.49	876.19	0.22	70.09	1884.8
0.9	3	72.44	231.79	536.01	535.97	4.31	932.61	886.24	0.12	55.97	1879.3

α=0.25

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	7	26.4	84.46	533.14	531.55	179.8	803.99	1081.7	5.41	131.69	2202.6
0.3	4	50.53	161.66	535.5	534.96	59.94	874.1	934.64	0.97	74.77	1944.4
0.6	4	53.34	170.67	565.34	565.21	14.27	923.57	884.63	0.22	70.77	1893.5
0.9	4	53.86	172.34	570.88	570.82	5.86	932.76	875.93	0.09	70.07	1884.7

θ=0.1129, 0.3387, 0.6774, 1.0161

Figure 4.11: Results when $\beta = 2.25, h_b = 5$, and *PM Ratio=1.0*

h_b=5 β=3.25

α=0.01	Expected Total Cost when			
μ	No Age Red	PM Ratio=0.1	PM Ratio=0.5	PM Ratio=1.0
0.1	1997.67	1977.61	1913.45	1896.54
0.3	1818.68	1823.36	1846.18	1876
0.6	1809	1815.93	1843.88	1874.93
0.9	1808.12	1815.25	1843.68	1874.84

α=0.05	Expected Total Cost when			
μ	No Age Red	PM Ratio=0.1	PM Ratio=0.5	PM Ratio=1.0
0.1	2332.34	2243.14	2010.15	1937.8
0.3	1849.14	1847.1	1853.66	1879.45
0.6	1812.65	1818.69	1844.69	1875.3
0.9	1809.12	1816.01	1843.9	1874.94

α=0.10	Expected Total Cost when			
μ	No Age Red	PM Ratio=0.1	PM Ratio=0.5	PM Ratio=1.0
0.1	2566.34	2426.15	2078.4	1964.28
0.3	1882.06	1873.43	1862.6	1882.47
0.6	1817.12	1822.1	1845.7	1875.76
0.9	1810.35	1816.94	1844.17	1875.06

α=0.25	Expected Total Cost when			
μ	No Age Red	PM Ratio=0.1	PM Ratio=0.5	PM Ratio=1.0
0.1	2976.41	2742.9	2197.17	2021.92
0.3	1960.99	1936.64	1887.29	1887.69
0.6	1829.92	1834.94	1848.7	1877.13
0.9	1814	1819.71	1844.98	1875.43

Figure 4.12: Comparison of different PM policies when $\beta = 3.25$ and $h_b = 5$

When No Age Reduction

For $\alpha = 0.01$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	$TotalCost$
0.10	6	31.71	101.44	538.92	538.16	85.20	33.37	2065.76
0.30	6	32.76	104.81	556.81	556.58	24.36	9.60	1979.11
0.60	5	38.86	124.35	536.26	536.15	12.10	4.09	1958.35
0.90	5	38.97	124.71	537.81	537.75	7.36	2.49	1951.97

For $\alpha = 0.05$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	$TotalCost$
0.10	9	19.98	63.91	531.25	528.75	283.67	35.33	2345.10
0.30	6	31.00	99.19	526.93	525.93	114.10	9.51	2073.48
0.60	6	32.23	103.11	547.79	547.32	51.10	4.13	2001.12
0.90	6	32.62	104.36	554.42	554.13	31.39	2.51	1979.19

For $\alpha = 0.10$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	$TotalCost$
0.10	11	15.51	49.58	511.35	507.57	446.57	35.91	2600.47
0.30	7	25.86	82.72	522.16	520.49	192.29	9.56	2173.50
0.60	6	31.28	100.08	531.70	530.83	98.58	4.10	2051.35
0.90	6	32.02	102.45	544.25	543.69	61.38	2.50	2010.23

For $\alpha = 0.25$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	$TotalCost$
0.10	16	10.05	32.11	491.70	485.72	738.56	36.74	3147.06
0.30	10	17.67	56.51	526.29	523.36	336.09	9.69	2410.72
0.60	8	23.15	74.06	541.60	539.93	186.18	4.14	2179.88
0.90	7	26.83	85.84	541.85	540.68	129.60	2.50	2096.27

Table 4.9: Results when $\beta = 1.25$, $h_b = 7$ and *No Age Reduction*

$\underline{When\ C_{pm}/C_{pm}^0 = 0.1}$

For $\alpha = 0.01$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	6	31.92	102.13	542.55	541.81	82.86	32.40	11.07	2072.91
0.30	5	38.70	123.83	534.04	533.80	27.18	9.18	9.37	1987.96
0.60	5	39.07	125.00	539.08	538.97	11.79	3.98	9.28	1967.14
0.90	5	39.16	125.31	540.38	540.32	7.16	2.42	9.25	1960.93

For $\alpha = 0.05$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	9	20.27	64.83	538.90	536.46	273.17	33.70	16.78	2345.08
0.30	6	31.27	100.01	531.32	530.34	110.87	9.20	11.31	2080.15
0.60	6	32.43	103.76	551.22	550.77	49.50	3.99	10.89	2009.92
0.90	6	32.83	105.04	558.05	557.76	30.41	2.43	10.76	1988.67

For $\alpha = 0.10$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	11	15.83	50.60	521.80	518.10	427.93	33.88	21.23	2590.38
0.30	7	26.13	83.58	527.61	525.97	186.14	9.19	13.31	2177.91
0.60	6	31.52	100.84	535.72	534.87	95.65	3.97	11.22	2058.63
0.90	6	32.24	103.13	547.90	547.20	59.49	2.42	10.96	2018.74

For $\alpha = 0.25$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	16	10.40	33.23	508.84	502.97	700.39	33.81	31.81	3107.09
0.30	10	17.97	57.46	535.15	532.29	321.91	9.16	18.79	2408.01
0.60	7	25.93	82.92	523.47	521.76	196.32	3.93	13.42	2184.74
0.90	7	27.09	86.66	547.08	545.94	125.25	2.41	12.82	2103.18

Table 4.10: When $\beta = 1.25$, $h_b = 7$ and PM ratio is 0.1

$\underline{When\ C_{pm}/C_{pm}^{0} = 0.5}$

For $\alpha = 0.01$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	6	32.77	104.83	556.90	556.22	72.99	28.55	53.94	2100.54
0.30	5	39.50	126.39	545.08	544.86	24.08	8.14	45.88	2020.24
0.60	5	39.84	127.47	549.72	549.63	10.40	3.52	45.49	2001.76
0.90	5	39.94	127.80	551.14	551.08	6.32	2.14	45.37	1996.27

For $\alpha = 0.05$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	8	23.21	74.19	542.56	540.26	255.24	28.26	74.04	2346.72
0.25	6	32.23	103.09	547.67	546.78	97.12	7.99	54.87	2105.84
0.50	5	38.90	124.45	536.69	536.24	50.53	3.50	46.62	2042.77
0.75	5	39.34	125.86	542.78	542.49	31.01	2.13	46.08	2021.30

For $\alpha = 0.10$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	10	18.03	57.63	536.71	533.25	388.71	27.72	93.76	2567.37
0.25	7	27.21	87.01	549.27	547.80	161.71	7.84	63.89	2195.55
0.50	6	32.48	103.90	551.98	551.22	83.58	3.43	54.43	2086.86
0.75	5	38.65	123.65	533.23	532.69	60.70	2.12	46.93	2052.01

For $\alpha = 0.25$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	15	11.92	38.08	545.08	539.56	613.46	26.47	139.00	3025.21
0.25	9	20.51	65.58	545.14	542.47	295.21	7.55	82.95	2403.71
0.50	7	27.05	86.50	546.06	544.51	170.67	3.35	64.28	2201.24
0.75	6	31.71	101.42	538.83	537.73	122.72	2.07	55.79	2127.76

Table 4.11: When $\beta = 1.25$, $h_b = 7$ and PM ratio is 0.5

When $C_{pm}/C^0_{pm} = 1.0$

For $\alpha = 0.01$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	5	39.50	126.36	544.93	544.32	67.36	23.77	91.86	2125.04
0.30	5	40.53	129.69	559.27	559.10	18.82	6.69	89.43	2058.10
0.60	5	40.78	130.49	562.73	562.65	8.08	2.87	88.86	2043.46
0.90	5	40.89	130.83	564.21	564.16	4.90	1.74	88.63	2039.13

For $\alpha = 0.05$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	7	27.17	86.87	548.39	546.34	225.02	23.03	128.12	2338.45
0.25	5	38.89	124.41	536.52	535.72	89.53	6.64	93.33	2131.56
0.50	5	40.05	128.14	552.60	552.24	39.53	2.86	90.54	2075.42
0.75	5	40.40	129.25	557.39	557.17	24.16	1.74	89.74	2058.58

For $\alpha = 0.10$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	9	20.92	66.85	555.69	552.59	336.73	22.32	162.87	2534.81
0.25	6	32.21	103.00	547.20	545.88	144.63	6.42	109.91	2206.81
0.50	5	39.17	125.30	540.36	539.66	76.94	2.85	92.65	2114.44
0.75	5	39.85	127.49	549.79	549.36	47.52	1.73	91.02	2082.55

For $\alpha = 0.25$

μ	n	q_1	q_2	Q_v	Q_b	EQC	ERC	PMC	$TotalCost$
0.10	12	14.93	47.67	539.32	534.21	574.58	21.29	224.63	2951.25
0.25	8	23.87	76.31	558.01	555.63	256.44	6.08	143.98	2389.91
0.50	6	32.04	102.44	544.24	542.86	152.62	2.74	110.53	2212.12
0.75	6	33.11	105.89	562.56	561.66	96.00	1.67	106.83	2149.27

Table 4.12: Results when $\beta = 1.25$, $h_b = 7$ and *PM ratio is* 1.0

Beta=2.25 $\underline{h_2=7}$

No Age Reduction

$\alpha=0.01$

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	TC
0.1	6	29.97	95.9	509.47	508.67	95.23	874.54	1081.3	39.21	2090.2
0.3	5	38.53	123.3	531.75	531.64	12.54	954.86	987.52	4.43	1959.3
0.6	5	39.03	124.88	538.56	538.54	2.76	967.25	974.87	0.97	1945.9
0.9	5	39.12	125.2	539.92	539.91	1.12	969.72	972.39	0.39	1943.6

$\alpha=0.05$

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	TC
0.1	7	21.19	67.81	428.05	425.94	296.06	709.51	1349.9	35.3	2390.8
0.3	6	31.29	100.12	531.88	531.42	52.04	913.7	1035	4.45	2005.2
0.6	5	38.56	123.4	532.18	532.06	13.43	955.62	986.73	0.96	1956.7
0.9	5	38.92	124.55	537.14	537.09	5.52	964.65	977.5	0.39	1948.1

$\alpha=0.10$

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	TC
0.1	8	16.52	52.86	386.53	383.82	424.02	624.33	1563.2	32.66	2644.3
0.3	6	29.87	95.59	507.8	507	94	871.69	1084.8	4.21	2054.7
0.6	5	38.01	121.64	524.58	524.36	26.01	941.79	1001.2	0.94	1970
0.9	5	38.67	123.73	533.6	533.5	10.88	958.21	984.06	0.39	1953.5

$\alpha=0.25$

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	TC
0.1	9	12.08	38.65	321.26	317.87	640.38	507.33	1966.2	27.21	3141.2
0.3	7	23.78	76.09	480.33	478.91	177.54	797.87	1200.7	3.96	2180
0.6	6	31.23	99.94	530.92	530.44	55.25	912.01	1036.9	0.95	2005.1
0.9	5	38.02	121.67	524.68	524.45	26.2	941.96	1001	0.38	1969.6

θ=0.1129, 0.3387, 0.6774, 1.0161

Figure 4.13: Results when $\beta = 2.25$,$h_b = 7$, and *No Age Reduction*

Beta=2.25 h_2=7

PM Ratio=0.1

α=0.01

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	6	30.48	97.53	518.14	517.45	80.94	889.65	1062.9	33.83	11.6	2078.9
0.3	5	38.81	124.17	535.48	535.39	10.51	961.6	980.6	3.75	9.34	1965.8
0.6	5	39.24	125.56	541.49	541.47	2.3	972.53	969.58	0.82	9.23	1954.5
0.9	5	39.31	125.79	542.48	542.47	0.93	974.33	967.79	0.33	9.22	1952.6

α=0.05

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	8	20.11	64.35	470.55	468.68	239.16	762.51	1280.2	30.64	17.07	2329.6
0.3	6	31.82	101.81	540.84	540.45	42.81	929.25	1017.7	3.66	11.1	2004.5
0.6	5	38.86	124.33	536.17	536.07	11.24	962.83	979.35	0.81	9.33	1963.5
0.9	5	39.15	125.27	540.23	540.19	4.6	970.21	971.89	0.33	9.26	1956.3

α=0.10

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	9	16.3	52.14	433.43	430.93	347.83	688.07	1450.3	27.63	20.88	2534.8
0.3	6	30.58	97.83	519.74	519.06	78.42	892.42	1059.6	3.49	11.56	2045.5
0.6	5	38.37	122.78	529.5	529.31	21.86	950.68	991.85	0.79	9.45	1974.6
0.9	5	38.94	124.61	537.38	537.3	9.09	965.03	977.11	0.33	9.31	1960.9

α=0.25

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	12	11.14	35.62	402.99	399.6	508.33	614.42	1751.8	23.64	30.03	2928.2
0.3	7	24.76	79.22	500.06	498.83	147.84	831.08	1152.7	3.22	14.03	2148.9
0.6	6	31.77	101.65	539.99	539.59	45.34	927.75	1019.3	0.78	11.12	2004.3
0.9	5	38.39	122.82	529.68	529.48	22.01	950.99	991.53	0.32	9.44	1974.3

θ=0.1129, 0.3387, 0.6774, 1.0161

Figure 4.14: Results when $\beta = 2.25, h_b = 7$, and *PM Ratio=0.1*

Beta=2.25 h_2=7

PM Ratio=0.5

α=0.01

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	6	32.51	104.03	552.68	552.32	38.71	949.65	995.79	17.45	54.32	2055.9
0.3	5	39.84	127.49	549.79	549.75	4.87	987.39	954.98	1.88	45.48	1994.6
0.6	5	40.04	128.13	552.57	552.56	1.04	992.43	950.13	0.4	45.24	1989.3
0.9	5	40.08	128.23	553	553	0.42	993.23	949.37	0.16	45.21	1988.4

α=0.05

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	7	25.94	82.99	523.85	522.73	128.63	870.85	1100	14.56	66.96	2181
0.3	5	39.03	124.88	538.53	538.32	23.28	966.85	975.26	1.83	46.44	2013.7
0.6	5	39.86	127.55	550.05	550	5.16	987.85	954.54	0.4	45.45	1993.4
0.9	5	40.01	128.01	552.05	552.03	2.09	991.49	951.03	0.16	45.29	1990.1

α=0.10

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	9	20.3	64.96	539.99	538.49	167.01	859.99	1160.7	12.01	83.57	2283.2
0.3	6	32.82	105.01	557.89	557.56	35.21	958.65	986.44	1.62	53.81	2035.7
0.6	5	39.63	126.81	546.85	546.76	10.18	982.02	960.2	0.4	45.72	1998.5
0.9	5	39.91	127.69	550.66	550.62	4.16	988.96	953.47	0.16	45.4	1992.2

α=0.25

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	11	15.85	50.69	522.76	520.51	258.89	808.85	1296.8	9.52	105.67	2479.7
0.3	6	31.27	100.07	531.61	530.91	79.07	912.77	1036	1.53	56.51	2085.8
0.6	5	39	124.78	538.1	537.88	24.56	966.06	976.06	0.39	46.48	2013.6
0.9	5	39.66	126.89	547.2	547.1	10.25	982.64	959.6	0.16	45.7	1998.3

θ=0.1129, 0.3387, 0.6774, 1.0161

Figure 4.15: Results when $\beta = 2.25, h_b = 7$, and *PM Ratio = 0.5*

Beta=2.25 h_2=7

PM Ratio=1.0

α=0.01

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	5	40.08	128.23	552.98	552.83	17.07	992.9	949.67	8.5	90.44	2058.6
0.3	5	40.92	130.93	564.64	564.63	1.55	1014.1	929.82	0.77	88.55	2034.8
0.6	5	40.98	131.13	565.5	565.5	0.33	1015.7	928.38	0.16	88.42	2033
0.9	5	40.99	131.16	565.63	565.63	0.13	1015.9	928.17	0.07	88.4	2032.7

α=0.05

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	6	32.85	105.09	558.31	557.78	56.81	958.95	986.06	6.89	107.57	2116.3
0.3	5	40.65	130.08	560.97	560.9	7.64	1007.4	936	0.76	89.14	2041
0.6	5	40.92	130.93	564.64	564.63	1.63	1014.1	929.82	0.16	88.55	2034.3
0.9	5	40.97	131.09	565.33	565.33	0.66	1015.4	928.67	0.07	88.44	2033.2

α=0.10

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	6	31.31	100.17	532.15	531.25	102.27	913.24	1035.3	6.51	112.94	2170.3
0.3	5	40.32	129.02	556.4	556.26	15.01	999.07	943.8	0.76	89.89	2048.5
0.6	5	40.85	130.72	563.71	563.68	3.26	1012.4	931.38	0.16	88.7	2035.9
0.9	5	40.93	130.98	564.85	564.84	1.31	1014.5	929.46	0.07	88.52	2033.9

α=0.25

μ	n	q_1	q_2	Q_1	Q_2	EQC	IHC	STC	ERC	PMC	TC
0.1	8	23.69	75.79	554.24	552.91	144.43	899.47	1085.2	4.82	144.69	2278.6
0.3	5	39.42	126.14	543.96	543.64	35.68	976.37	965.72	0.74	91.97	2070.5
0.6	5	40.63	129.99	560.6	560.53	8.04	1006.7	936.62	0.16	89.2	2040.8
0.9	5	40.86	130.73	563.75	563.72	3.27	1012.5	931.31	0.07	88.7	2035.8

θ=0.1129, 0.3387, 0.6774, 1.0161

Figure 4.16: Results when $\beta = 2.25, h_b = 7$, and *PM Ratio = 0.1*

h_b=7 β=3.25

α=0.01	Expected Total Cost when			
μ	No Age Red	PM Ratio=0.1	PM Ratio=0.5	PM Ratio=1.0
0.1	2112.21	2086.67	2028	2041.1
0.3	1950.81	1957.48	1979.24	2032.73
0.6	1943.05	1951.99	1987.96	2032.51
0.9	1942.36	1951.51	1987.85	2032.49

α=0.05	Expected Total Cost when			
μ	No Age Red	PM Ratio=0.1	PM Ratio=0.5	PM Ratio=1.0
0.1	2409.61	2311.44	2092.96	2060.78
0.3	1974.4	1974.39	1993.17	2033.34
0.6	1945.79	1953.9	1988.38	2032.57
0.9	1943.1	1952.02	1997.4	2032.5

α=0.10	Expected Total Cost when			
μ	No Age Red	PM Ratio=0.1	PM Ratio=0.5	PM Ratio=1.0
0.1	2628.74	2479.43	2155.63	2081.52
0.3	2000.76	1993.81	1997.97	2034.09
0.6	1949.15	1953.26	1988.9	2032.65
0.9	1944.02	1952.66	1988.1	2032.53

α=0.25	Expected Total Cost when			
μ	No Age Red	PM Ratio=0.1	PM Ratio=0.5	PM Ratio=1.0
0.1	3024.72	2781.06	2258.78	2121.19
0.3	2066.14	2040	2011.74	2036.34
0.6	1958.93	1963.17	1990.46	2032.89
0.9	1946.76	1954.57	1988.52	2032.59

Figure 4.17: Comparison of different PM policies when $\beta = 3.25$ and $h_b = 7$

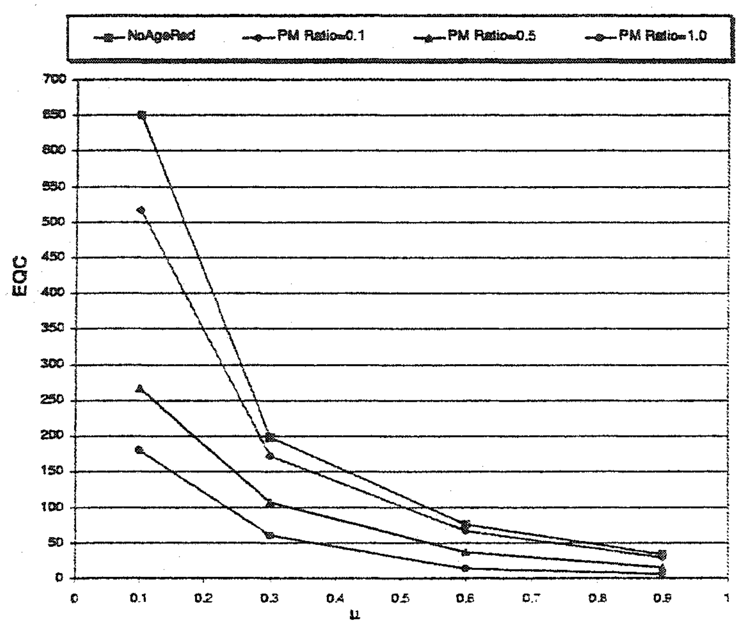

Figure 4.18: Graph Between μ and EQC, when $\beta = 2.25$ and $h_b = 5$, for different PM Policies

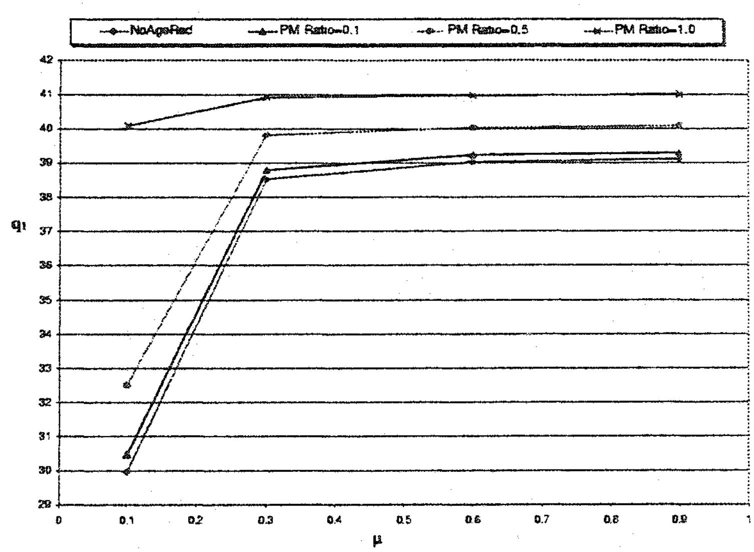

Figure 4.19: Graph between μ and q_1, when $\beta = 2.25$ and $h_b = 7$, for different PM policies

Chapter 5

Continuous review inventory policy with stochastic demand

In this chapter we will discuss the model in which explicit account is taken of the fact that demands on the system cannot be predicted with certainty but instead must be described probabilistically. The introduction of randomness into the model nature of the demand pattern brings to the forefront several new considerations which did not enter into the analysis when studying the deterministic models.

5.1 Overview

One of these new considerations concerns how much is known about the state of the system at any point in time. For the deterministic models, it is possible to determine for all future times precisely what the state of the system will be if the state is known at a given time and if the quantity to be ordered and the reorder point are specified. However, when randomness is introduced into the demand pattern, it is no longer possible to make such predictions, since the times of occurrence of the demands are random variables.

We will now discuss in detail the model given by BenDaya et. al [3], as this model has been extended to incorporate the quality and restoration feature.

5.1.1 Description of the Basic Model

In [3], it is assumed that the buyer used a (s, Q) inventory policy, where the lead time is considered as a function of the production lot size i.e. the lead time is proportional to the lot size produced by the vendor in addition to a fixed delay due to transportation, non productive time, etc. They also assumed that n equal shipments are made to the buyer by the vendor.

The notations and assumptions used for the model are as follows;

Notations

P = Production rate.

p = 1/P.

D = Demand rate.

q = Size of the batch produced by the vendor.

n = Number of shipments that the buyer received.

Q = Lot size received by the buyer or the lot size produced by the vendor,

= nq.

A = Ordering cost incurred by the buyer for the lot of size Q.

K = Setup cost incurred by the vendor.

F	=	Transportation cost incurred by the buyer with each shipment of size q.
s	=	Reorder point.
π	=	Cost due to lost sales incurred by the buyer.
$L(q)$	=	Lead time = $pq + b$.
h_b	=	Buyer's inventory holding cost.
h_v	=	Vendor's inventory holding cost.
ETC	=	Expected Total Cost,
	=	Holding cost for both vendor and buyer + ordering, transportation and back ordering cost for buyer + setup cost for vendor.

Assumptions

1. The buyer's inventory cost is always greater then the vendor's inventory cost.
2. Inventory is continuously reviewed and replenishment are made whenever the inventory level falls to the reorder points s.
3. The demand during the lead time follows normal distribution with mean $DL(q)$ and standard deviation $\sigma\sqrt{L(q)}$

The relationship between vendor and buyer can be described as follow; The buyer orders a lot of size nq and incurs an ordering cost A. The vendor manufactures the product in lots of size nq with a finite rate P and incurs a set up cost K. The buyer receives n lots of size q. He incurs a transportation cost F with each shipment of size q. The buyer places his order when his on hand inventory reaches a reorder

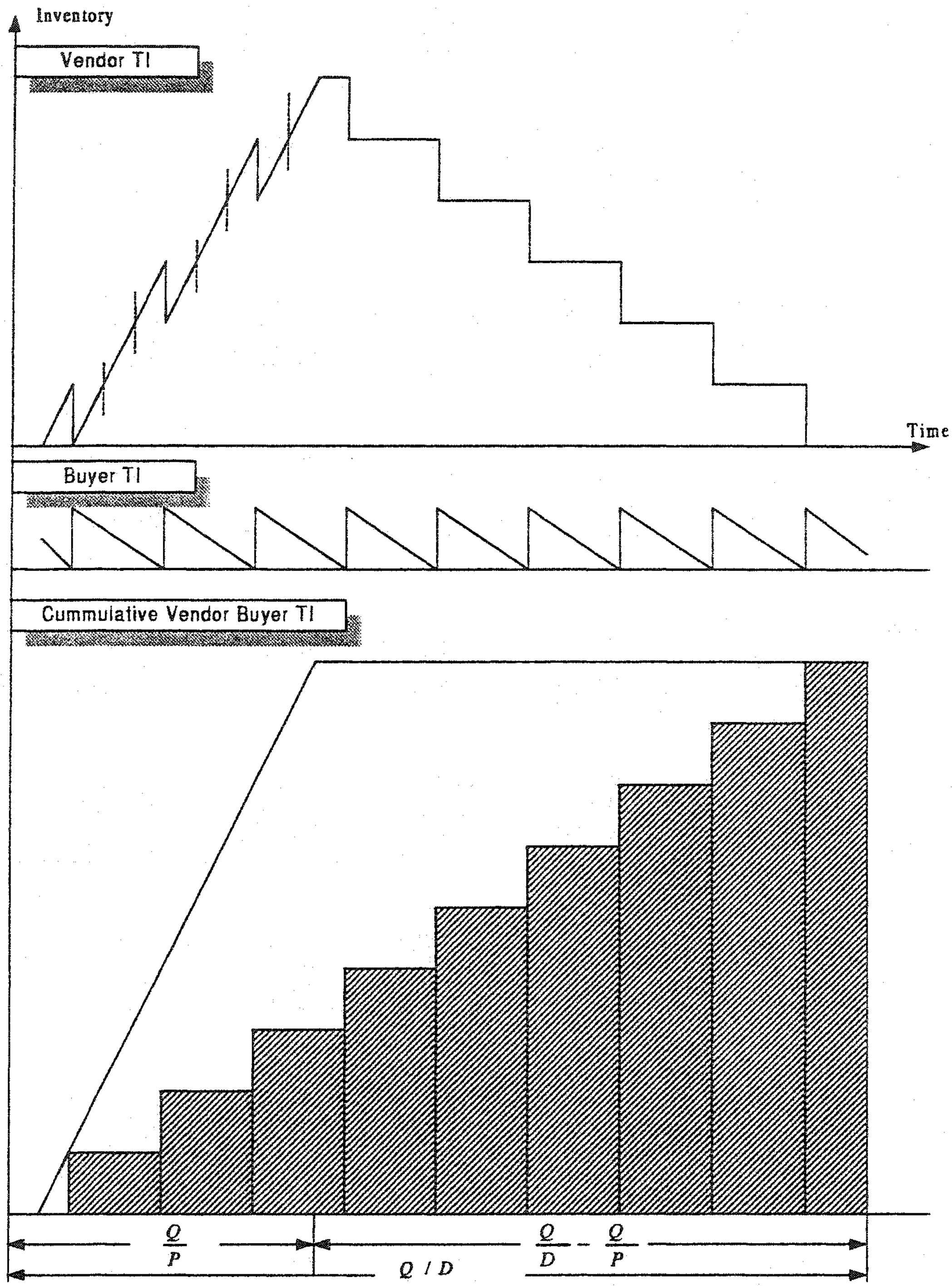

Figure 5.1: The inventory profile of combine vendor buyer system with perfect production process (when $n = 9$).

point s after receiving the n^{th} shipment. The inventory profile of both parties are depicted in the fig (5.1). The total expected cost for the system is given by;

$$\begin{aligned} ETC(q,s,n) &= \frac{D}{q}\left[F+\frac{A+K}{n}+\pi b\big(s,L(q)\big)\right]+\frac{q}{2}\left[h_b+h_v\{n\big(1-Dp\big)-1+2Dp\}\right] \\ &+ h_b\sigma k\sqrt{pq+b} \end{aligned} \tag{5.1}$$

where, $b(s,L(q))=\sigma\sqrt{pq+b}\;\psi(k)$, and

$$\begin{aligned} k &= \frac{s-DL(q)}{\sigma\sqrt{L(q)}} \\ \psi(k) &= \int_k^{\infty}(z-k)\phi(z)dz \end{aligned} \tag{5.2}$$

The problem is to find the number of shipment n, the shipment size q, and the reorder point s, that minimizes the expected total cost equation (5.1).

5.2 Integrated Imperfect Process Model

In this section it is assumed that the process starts in the in-control state and after sometimes shifts to an out of control state during a production run. The time to shift is assumed to follow exponential distribution with mean θ. The process is restored after producing each batch, if it is found in out of control state. The nonconforming items are screened out before a shipment is made.

5.2.1 Problem Definition

Since the model in [3], considered perfect production process, which is not practical in real life. Therefore it is assumed that the process starts in the in-control state then shifts to an out of control state at a random time. Then, it starts producing nonconforming items, until it is restored and brought back to the in control state. This restoration work is performed after completion of batches if the process is found to be in out of control state. The nonconforming items are screened out before shipment is made to the buyer and whenever the production stops. The total cost will be the sum of the cost due to nonconforming items, the inventory holding cost, setup and shipment cost, the restoration cost and the shortage cost. *For simplicity it is assumed that the lead time is constant and is independent of the production lot size.*

The **notations** used are:

q	=	Size of the batch produced by the vendor.
μ	=	Mean demand during lead time.
N_j	=	Number of non conforming items produced in the batch j.
g	=	The shipment size received by the buyer, $= q - N_1$.
L	=	Lead time.
Q_v	=	The lot size produced by the vendor. $= nq$.
Q_b	=	The lot size received by the buyer. $= ng$.
EQC	=	Expected Quality Cost.
ERC	=	Expected Restoration Cost.
ETC	=	Expected total cost.

Assumptions

It is assumed that the process starts in the in control state and after some time shifts to an out of control state where it starts producing a fraction of nonconforming items. The process is restored after producing each batch and if it is found to be out of control. The inspection of the product is done before shipping it to the buyer, and is assumed to be error free. The time taken by inspection and restoration is negligible. It is also assumed that the nonconforming items produced in any interval are proportional to their time. The safety stock which the buyer have at the start of the cycle is $\frac{gD}{P}$. The inventory profile of the vendor and buyer after integration is shown in Figure 5.2.

Calculating Non-conforming items

It is assumed that the production process, after shifting to an out of control state, starts producing a fixed fraction α of nonconforming items. This process shift is assumed to follow exponential distribution with mean θ, so the nonconforming items produced in a batch will be calculated from;

$$N_j = \int_0^{t_j} \alpha P(t_j - t) f_j(t) dt \quad for j = 1, \cdots, n \tag{5.3}$$

where, $f_j(t) = \frac{1}{\theta} e^{\frac{-t}{\theta}}$ and t_j is the production time of batch j.

Let N_1 is the number of nonconforming items produced in the batch q, then the quantity of good items produced i.e. g or in other words the size of the batch shipped to the buyer will be;

$$g = q - N_1 \tag{5.4}$$

The lot size produced by the vendor is

$$Q_v = nq \tag{5.5}$$

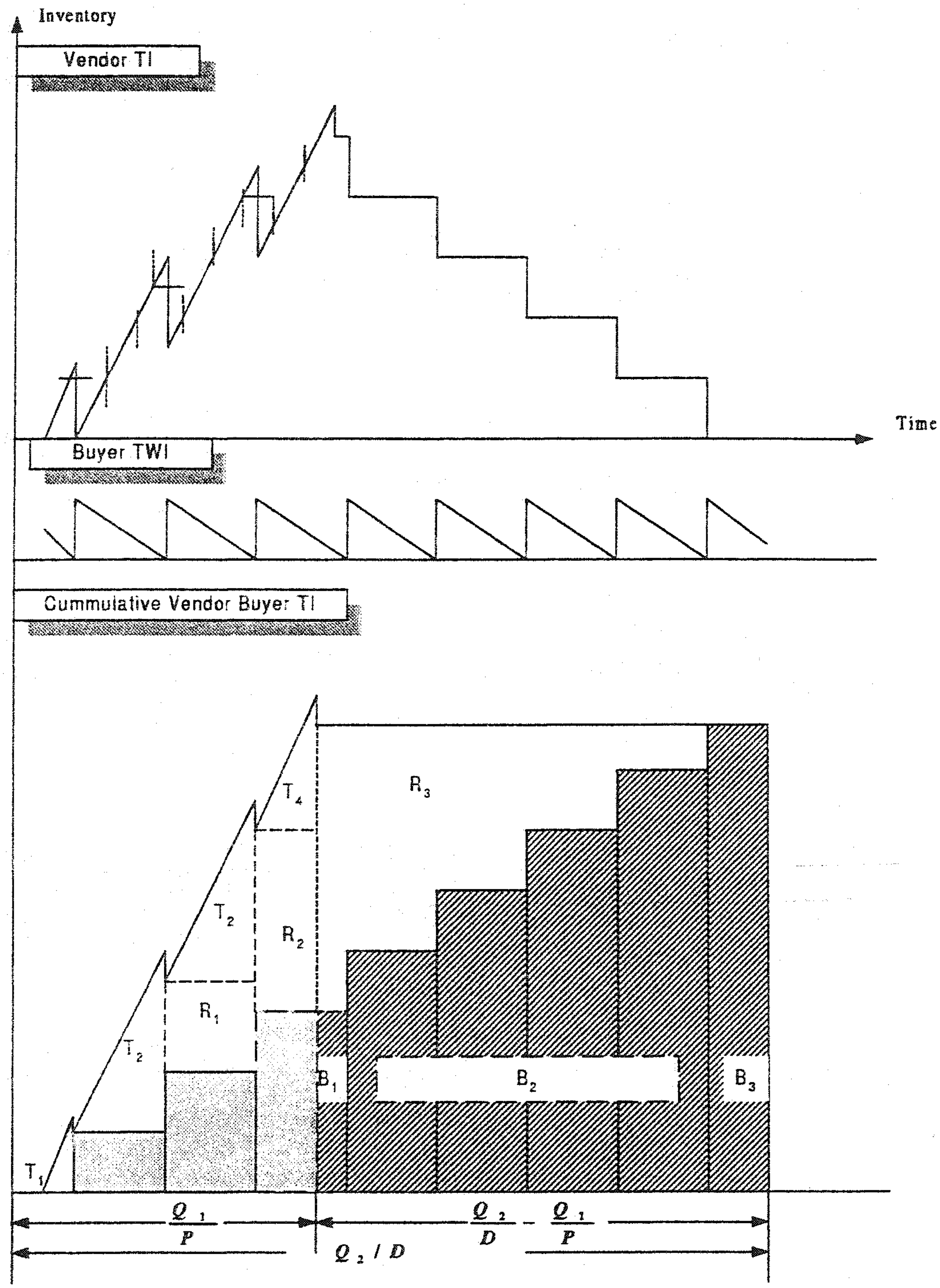

Figure 5.2: The inventory profile of combine vendor buyer system with imperfect process (when $n = 8$).

hence, the lot size that will be shipped to the buyer will be;

$$Q_b = ng \tag{5.6}$$

Let k' represent the number of batches that are going to be produced during the interval g/D, which is;

$$k' = \frac{g/D}{q/P}$$

From eq. (5.3) we can determine the nonconforming items produced during different interval. i.e.

$$\begin{aligned} N_j &= \alpha P\left[\frac{q}{P} - \theta + \theta exp\left(-\frac{q}{P\theta}\right)\right] \quad \text{for } j = 1, \cdots, n \\ \eta_{k'} &= k'\left[\alpha P\left(\frac{q}{P} - \theta + \theta exp\left(-\frac{q}{P\theta}\right)\right)\right] \end{aligned} \tag{5.7}$$

Where, $\eta_{k'}$ represents the number of nonconforming items produced during the consumption of batch of size g.

5.2.2 The Total Expected Cost

The total cost will be the sum of the cost incurred by the buyer and the vendor.

Buyer's Cost

Ordering and Transportation Cost

The Total ordering and transportation cost per unit time will be

$$\begin{aligned} &= \frac{D}{Q_b}(A + nF) \\ &= \frac{D}{ng}(A + nF) \end{aligned} \tag{5.8}$$

Inventory Holding Cost

The average inventory for the buyer will be

$$\bar{I} = \frac{g}{2} + (s - \mu) \tag{5.9}$$

and the inventory holding cost incurred by the buyer is;

$$IHC_b = h_b \left(\frac{g}{2} + (s - \mu) \right) \tag{5.10}$$

Back Ordering Cost

Since it is assumed that the demand during the lead time is normally distributed with mean DL and standard deviation $\sigma\sqrt{L}$. Since the expected number of shortages $= b(s, L) = \int_s^\infty (x - s) f(x, \mu, \sigma) dx$, where, $\mu = DL$, $\sigma = \sigma\sqrt{L}$ and x is the demand during lead time.

Therefore the cost due to shortages will be

$$\frac{\pi D}{g} b(s, L) \tag{5.11}$$

Now the total cost incurred by the buyer will be the sum of equations (5.8), (5.10) and (5.11).

$$TC_b = (A + nF)\frac{D}{ng} + h_b \left[\frac{g}{2} + (s - \mu) \right] + \frac{\pi D}{g} b(s, L) \tag{5.12}$$

Vendor's Cost

The cost incurred by vendor will be the sum of setup cost and the inventory holding cost.

Setup Cost

The setup cost per unit time will become;

$$\begin{aligned} &= \frac{KD}{Q_b} \\ &= \frac{KD}{ng} \end{aligned} \tag{5.13}$$

Inventory Holding Cost

Let TI_v denotes the Total Inventory of vendor, then

$$IHC_v = \frac{h_v D}{ng} TI_v \tag{5.14}$$

Total Inventory For the Vendor

Considering Figure 5.2, let i be the smallest integer such that

$$i\left(\frac{g}{D}\right) + \frac{q}{P} \geq \frac{Q_v}{P}$$

As $\eta_{k'}$ represent the nonconforming items produced during interval $\frac{g}{D}$ i.e. $\eta_{k'} = k'N_1$, where k' is the ratio between g/D and q/P, and Let τ is the time interval such that;

$$\tau = \frac{Q_v}{P} - \left[(i-1)\frac{g}{D} + \frac{q}{P}\right]$$

From Figure 5.2, the total inventory of vendor can be find out by summing the areas of different triangles and rectangles.

$$\begin{aligned} T_1 &= \frac{q^2}{2P} \\ T_2 &= (i-1)\frac{P}{2}\left(\frac{g}{D}\right)^2 \\ T_3 &= \frac{P}{2}(\tau)^2 \\ R_1 &= \frac{g}{D}\sum_{j=1}^{i-2} j\left(P\frac{g}{D} - \eta_{k'} - g\right) \quad \text{if } i \geq 3 \end{aligned}$$

$$\begin{aligned}
R_2 &= (i-1)\left[P\frac{g}{D} - \eta_{k'} - g\right](\tau) \\
R_3 &= Q_b\left[\frac{Q_b}{D} - \frac{Q_v}{P}\right] \\
B_1 &= (ig)\left[\frac{ig}{D} + \frac{q}{P} - \frac{Q_v}{P}\right]
\end{aligned}$$

if $n - i = 2$, then

$$B_2 = (n-1)\frac{g^2}{D}$$

if $n - i > 2$, then

$$\begin{aligned}
B_2 &= \sum_{j=i+1}^{n-1} j\frac{g^2}{D} \\
B_3 &= Q_b\left[\frac{g}{D} - \frac{q}{P}\right] \qquad (5.15)
\end{aligned}$$

Now TI_v will be;

$$TI_v = \sum_{q=1}^{3} T_q + \sum_{r=1}^{3} R_r - \sum_{s=1}^{3} B_s \qquad (5.16)$$

Cost Due To Defective Items

The quality cost incurred by the vendor because of producing the bad items will be;

$$EQC = DC\frac{D}{Q_b}(Q_v - Q_b) \qquad (5.17)$$

Where, DC is the cost of producing a defective item.

Restoration Cost

Since restoration is performed after producing each batch and if the process is found out to be in out of control state,so the restoration cost will become;

$$ERC_j = \int_0^{t_j} \Big(r_0 + r_1\big(t_j - t\big)\Big)\frac{1}{\theta}e^{-t/\theta}dt$$

The expected restoration cost for the whole system is simply;

$$ERC = \frac{D}{Q_b}\sum_{j=1}^{n}\left[\left(r_0 + r_1\frac{q}{P} - r_1\theta\right) + \left(r_1\theta - r_0\right)e^{\frac{-q}{P\theta}}\right] \tag{5.18}$$

Total cost incurred by the vendor

Hence the total cost incurred by the vendor will be the sum of equations (5.13),(5.14),(5.17) and (5.18) ;

$$TC_v = \frac{KD}{ng} + \frac{h_v D}{ng} TI_v + EQC + ERC \tag{5.19}$$

The total cost of the system will be the sum of equations (5.12) and (5.19) i.e.

$$\begin{aligned} ETC(g,s,n) &= (A+nF)\frac{D}{ng} + h_b\left[\frac{g}{2} + (s-\mu)\right] + \frac{\pi D}{g}\, b(s,L) \\ &+ \frac{KD}{ng} + \frac{h_v D}{ng} TI_v + EQC + ERC \end{aligned} \tag{5.20}$$

Since it is assumed that the demand during lead time is normally distributed with mean DL and standard deviation $\sigma\sqrt{L}$. In this case $b(s,L) = \sigma\sqrt{L}\psi(k)$ where,

$$\begin{aligned} k &= \frac{(s-DL)}{\sigma\sqrt{L}} \\ \psi(k) &= \int_k^{\infty}(z-k)f(z)dz \end{aligned}$$

and $f(z)$ is the standard normal probability density function. After making these changes, equation (5.20) can be written as;

$$\begin{aligned} ETC(g,k,n) &= (A+nF)\frac{D}{ng} + h_b\left[\frac{g}{2} + k\sigma\sqrt{L}\right] + \frac{\pi D}{g}\,\sigma\sqrt{L}\psi(k) \\ &+ \frac{KD}{ng} + \frac{h_v D}{ng} TWI_v + EQC + ERC \end{aligned} \tag{5.21}$$

For a fixed n, taking the derivative with respect to k.

$$\begin{aligned} \frac{\partial ETC}{\partial k} &= h_b\, \sigma \sqrt{(L)} + \frac{\pi D}{g}\, \sigma \sqrt{(L)}[-\bar{F}(k)] = 0 \\ \bar{F}(k) &= \frac{g\, h_b}{\pi D} \end{aligned} \tag{5.22}$$

Numerical Example

The data are shown in table (5.1).

K=400	,	F=25 .
h_v=4	,	h_b= 5.
P=3200	,	D = 1000.
L=0.1	,	σ=5.
π = 100	,	DC = 30.
r_0 = 4	,	r_1 = 2.

Table 5.1: Problem Parameters

After making the new assumption in the basic model, that the lead time is no longer the function of production lot size. The results are summarized in the table (5.2).

We employ the Hooke and Jeeves [60] pattern search technique to determine the batch size q and number of shipments n, after varying the percent nonconforming items α and time to shift θ. The results obtained after integrating the quality and restoration aspects are summarized in Tables 5.3, 5.4 and 5.5. Table 5.3 summarizes the results obtained for the case when $F = 15$, whereas Table 5.4 and 5.5 shows the results for $h_b = 7$ & 10 respectively.

The number of batches, the size of batches, the expected quality and total cost per unit time are sensitive to the proportion of nonconforming items produced, as it can be seen from Figures 5.3, 5.4, 5.5 and 5.6 respectively. So for high proportion of nonconforming items, we have to divide the lot in to more batches of smaller sizes. The total cost will also be high when producing high number of nonconforming items.

5.2.3 Conclusion

In this chapter we incorporated the quality and restoration aspects in integrated vendor buyer model, where the buyer faces stochastic demand. The products are screened out before a shipment is made and the process is restored after producing each batch if it is found in out of control state. Numerical examples have been presented to illustrate important features of the proposed model.

For Different Values of F

F	n	q	Q	ROL	$Total\ Cost$
35	4	143	572	11	2076.19
25	5	116	580	11	2000.54
15	6	95	570	11	1906.84

For Different Values of h_b

h_b	n	q	Q	ROL	$Total\ Cost$
5	5	116	580	11	2000.54
7	6	96	576	11	2109.09
10	7	80	560	11	2241.38

For Different Values of L

L	n	q	Q	ROL	$Total\ Cost$
0.05	5	116	580	53	2007.96
0.01	5	116	580	11	2000.54
0.005	5	115	575	6	1997.09

For Different Values of π

π	n	q	Q	ROL	$Total\ Cost$
150	5	116	580	11	2002.37
100	5	116	580	11	2000.54
50	5	116	580	11	1998.71

Table 5.2: Effect on different parameters for perfect Production Processes

When $F = 15$

For $\alpha = 0.01$

θ	n	q	ROL	Q_v	Q_b	QC	RC	$Total\ Cost$
0.01	7	81.70	11.3	572.0	568.3	192.9	45.8	2146.3
0.05	7	81.70	11.3	571.9	570.6	65.20	19.8	1992.3
0.10	7	82.20	11.3	575.2	574.5	35.50	11.1	1953.6
0.50	6	94.60	11.3	567.5	567.3	8.70	2.40	1915.8
1.00	6	94.80	11.3	568.9	568.8	4.04	1.20	1910.3

For $\alpha = 0.05$

θ	n	q	ROL	Q_v	Q_b	QC	RC	$Total\ Cost$
0.01	17	34.30	11.5	583.1	571.8	591.1	78.4	2750.7
0.05	11	52.40	11.4	576.7	572.5	222.6	21.6	2203.0
0.10	9	63.80	11.4	573.9	571.3	140.7	11.4	2079.5
0.50	7	82.0	11.3	574.0	573.3	37.80	2.50	1947.4
1.00	7	82.60	11.3	578.4	578.0	19.2	1.20	1927.3

For $\alpha = 0.1$

θ	n	q	ROL	Q_v	Q_b	QC	RC	$Total\ Cost$
0.01	27	21.8	11.5	589.0	572.8	848.30	93.4	3259.2
0.05	15	38.8	11.4	582.1	575.5	339.9	22.5	2398.9
0.10	12	48.7	11.4	584.1	579.9	218.6	11.7	2204.9
0.50	8	72.40	11.3	579.6	578.3	67.10	2.5	1984.0
1.00	7	82.0	11.3	573.8	573.1	38.1	1.20	1946.5

Table 5.3: Effect on different parameters for imperfect Production Processes (when $F = 15$)

When $h_b = 7$

For $\alpha = 0.01$

θ	n	q	ROL	Q_v	Q_b	QC	RC	$Total\ Cost$
0.01	7	83.50	11.3	584.3	580.6	194.7	45.1	2352.8
0.05	7	83.10	11.3	581.6	580.3	66.10	19.7	2198.7
0.10	6	93.80	11.2	562.9	562.2	40.00	10.9	2158.7
0.50	6	95.00	11.2	569.8	569.6	8.70	2.40	2118.6
1.00	6	95.30	11.2	571.7	571.6	4.40	1.20	2113.1

For $\alpha = 0.05$

θ	n	q	ROL	Q_v	Q_b	QC	RC	$Total\ Cost$
0.01	13	45.60	11.4	593.2	579.3	717.6	68.5	3048.2
0.05	9	63.80	11.3	574.5	569.5	265.4	20.9	2437.4
0.10	8	72.20	11.3	577.5	574.7	158.0	11.3	2297.5
0.50	6	93.6	11.2	561.4	560.6	43.10	2.40	2153.4
1.00	6	94.40	11.2	566.2	565.8	21.9	1.20	2130.7

For $\alpha = 0.1$

θ	n	q	ROL	Q_v	Q_b	QC	RC	$Total\ Cost$
0.01	20	29.6	11.4	591.9	571.3	1080.2	84.7	3693.3
0.05	12	48.6	11.4	583.5	575.5	418.8	21.9	2674.7
0.10	10	58.7	11.3	586.6	581.7	261.2	11.6	2444.7
0.50	7	82.40	11.3	576.5	575.1	76.10	2.5	2192.0
1.00	6	93.5	11.2	561.1	560.3	43.5	1.20	2152.6

Table 5.4: Effect on different parameters for imperfect Production Processes (when $h_b = 7$)

<u>*When* $h_b = 10$</u>

For $\alpha = 0.01$

θ	n	q	ROL	Q_v	Q_b	QC	RC	$Total\ Cost$
0.01	8	71.90	11.2	574.9	571.4	181.6	50.4	2471.9
0.05	8	71.80	11.2	574.1	573.0	58.40	20.3	2318.9
0.10	8	72.20	11.2	577.9	577.3	31.50	11.3	2282.9
0.50	8	72.80	11.2	582.7	582.5	6.70	2.50	2249.3
1.00	7	80.00	11.2	559.8	559.7	3.70	1.20	2244.7

For $\alpha = 0.05$

θ	n	q	ROL	Q_v	Q_b	QC	RC	$Total\ Cost$
0.01	14	42.30	11.3	591.7	578.5	682.6	71.2	3117.2
0.05	10	57.30	11.3	572.6	568.1	241.0	21.3	2529.8
0.10	9	63.80	11.2	573.8	571.1	140.6	11.4	2402.2
0.50	8	72.1	11.2	576.5	575.8	33.30	2.50	2276.0
1.00	8	72.60	11.2	580.8	580.4	16.9	1.20	2258.2

For $\alpha = 0.1$

θ	n	q	ROL	Q_v	Q_b	QC	RC	$Total\ Cost$
0.01	21	28.5	11.4	597.5	577.3	1048.1	85.9	3739.2
0.05	13	45.1	11.3	585.7	578.2	390.4	22.2	2748.7
0.10	11	53.2	11.3	585.7	581.1	238.2	11.7	2533.8
0.50	8	71.20	11.2	569.4	568.1	65.90	2.5	2309.0
1.00	8	72.1	11.2	576.6	576.0	33.6	1.20	2275.0

Table 5.5: Effect on different parameters for imperfect Production Processes (when $h_b = 10$)

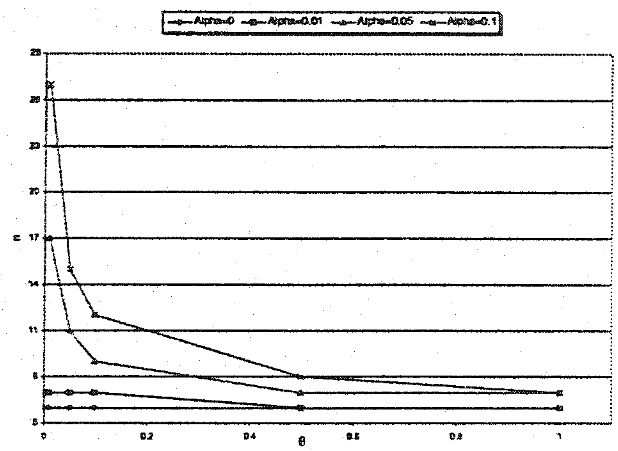

Figure 5.3: Effect of θ on n when $F = 15$ for (s, Q) policy

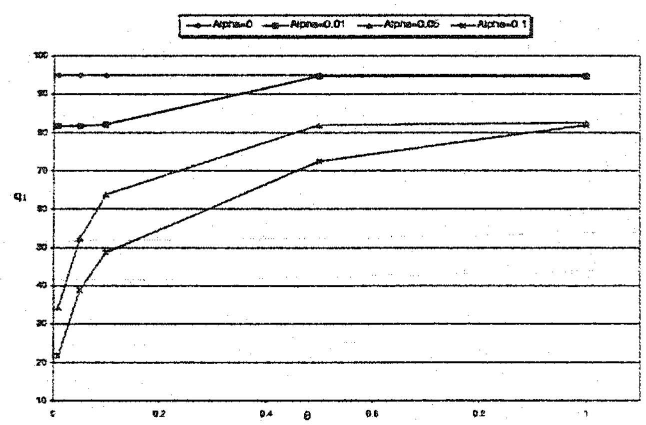

Figure 5.4: Effect of θ on q_1 when $F = 15$ for (s, Q) policy

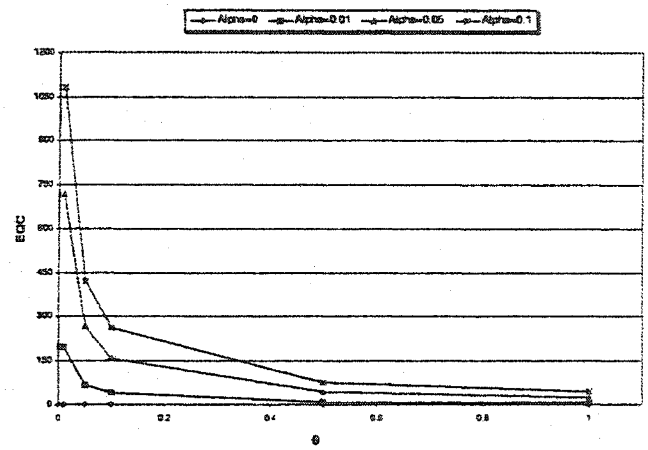

Figure 5.5: Effect of θ on EQC when $h_b = 7$ for (s, Q) policy

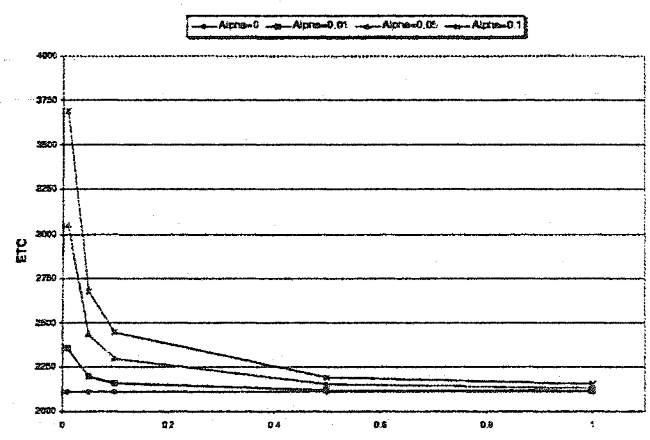

Figure 5.6: Effect of θ on ETC when $h_b = 7$ for (s, Q) policy

Chapter 6

Conclusion

6.1 Summary

In this thesis we developed and test generalized integrated mathematical models incorporating quality and maintenance with the number of shipments and lot sizing decisions. This has been done in the context of multi stage production inventory systems where batches are allowed to be shipped between stages during production and integrated vendor buyer problems where buyer faces both deterministic and stochastic demand. It was assumed that the screening of nonconforming items is done before shipment is made to the next stage or to the buyer and the restoration or maintenance work is performed after producing each batch. Numerical results are obtained for the sake of comparison and analysis, which demonstrates the usefulness of the approaches considered. We used Hooke and Jeeves optimization procedure for solving these models. Optimal number of shipments and lot sizes investment in the process of restoration and preventive maintenance is determined which minimizes the expected total cost per unit time. The results indicate the extent to which imperfectness of production processes, maintenance and process restoration can affect the lot sizing decision. The following conclusion can be drawn.

1. The number of shipment increases after integrating quality and restoration aspects, since it is assumed that restoration is performed after producing each batch.

2. The size of batches is reduced after incorporation, as it is better to produced less items when having a process shift.

3. After introducing preventive maintenance the quality cost decreases because of the fact that PM increases the time to shift of a process to an out of control state.

These models are more realistic since they consider quality in production planning decisions.

6.2 Contributions

This thesis contributes the following to the multistage lot sizing models with batch shipment.

1. Mathematical models for determining number of batches and lot sizes for multi-stage production inventory system with batch shipments are formulated incorporating.

    - Imperfect quality of production.

    - Process restoration is done after producing each batch if the process is found out to be in out of control state.

- The nonconforming items are screened out before the shipment is made to the next stage.

2. Mathematical models for determining number of batches and lot sizes for integrated vendor buyer system with deterministic demand rate are formulated incorporating.

   - Process restoration
   - Screening of nonconforming items before shipment to the buyer or whenever the production stops.
   - Imperfect production quality.
   - Preventive Maintenance.

3. Mathematical models for determining number of batches and lot sizes for integrated vendor buyer system where the buyer uses continuous review inventory policy and faces stochastic demand.

   - Imperfect production quality.
   - Process restoration after producing the batch and if the process is found out to be in out of control state.
   - Product screening of nonconforming items before shipment.

6.3 Models utilization

It has been mentioned earlier that the inventory at different stages results in unnecessary cost and because of this more and more companies or vendors are opting for Just In Time (JIT) production system, where batches are allowed to be transferred to the next stage before the completion of the whole lot. For that a number of models have been developed considering different shipping patterns, but all of them consider perfect production processes, which may not always be the case in real life. So in this thesis we integrate quality, maintenance and restoration features in some recent multistage production inventory models, in order to bring them closer to real life scenario.

6.4 Future Research

The models presented in this thesis can be extended by incorporating several real life considerations including the following;

- Effect of inspection errors while screening the nonconforming items before passing it to the next stage or to the buyer.
- Effect of errors committed in process inspection, preventive maintenance activities and process restoration.
- Considering the effect of batch transportation time, time taken for inspection and restoration, constraints on batch sizes and number of batches.
- Integrating transportation cost as a function of batch size in these models.

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